Section 10.2 Sinusoidal Sources

P10.2-1

Given:

$$v_1(t) = 5\cos\left(150t + 30^\circ\right)\text{ V}$$

$$v_2(t) = 4\cos\left(150t - 60^\circ\right)\text{ V}$$

So,

$$\theta_1 = 30^\circ$$

$$\theta_2 = -60^\circ$$

The period of the sinusoids is given by,

$$150 = \frac{2\pi}{T}$$

$$T = 42\text{ ms}$$

Compare $v_2(t)$ to $v_1(t)$

$$\theta_2 - \theta_1 = -60 - 30 = -90^\circ = -\frac{\pi}{2}\text{rad}$$

The minus sign (–) indicates a delay rather than an advance,
Convert the angle to time.

$$\theta_2 - \theta_1 = 2\pi\frac{t_d}{T}$$

$$t_d = \frac{(\theta_2 - \theta_1)T}{2\pi}$$

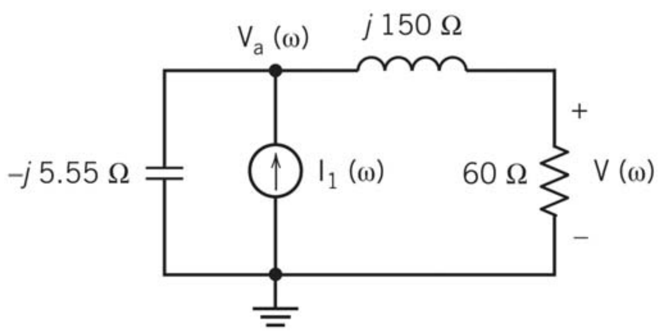
$$= \frac{(-\pi/2)\,(0.042)}{2\pi}$$

$$= -0.0105s$$

P10.4-5

Solution:

The figure below shows the given circuit:



The impedance of inductor is:

$$\mathbf{Z}_{\mathrm{L}} = j\omega L$$

Substitute $6000\ \mathrm{rad/s}$ for ω, and $25\ \mathrm{mH}$ for L,

$$\begin{aligned}\mathbf{Z}_{\mathrm{L}} &= j\left(6000\ \mathrm{rad/s}\right)\left(25\ \mathrm{mH}\right)\left(\frac{10^{-3}\ \mathrm{H}}{1\ \mathrm{mH}}\right)\\ &= j150\ \Omega\end{aligned}$$

The impedance of the capacitor is:

$$\mathbf{Z}_{\mathrm{C}} = \frac{-j}{\omega C}$$

Substitute $6000\ \mathrm{rad/s}$ for ω, and $30\ \mu\mathrm{F}$ for C,

$$\begin{aligned}\mathbf{Z}_{\mathrm{C}} &= \frac{-j}{\left(6000\ \mathrm{rad/s}\right)\left(30\ \mu\mathrm{F}\right)\left(\frac{10^{-6}\ \mathrm{F}}{1\ \mu\mathrm{F}}\right)}\\ &= -j5.55\ \Omega\end{aligned}$$

The phasor representation of current $i(t) = 180\cos(6000t)$ is:

$$\begin{aligned}\mathbf{I}(\omega) &= 180\angle 0^\circ\\ &= 180\cos(0^\circ) - j180\sin(0^\circ)\\ &= 180\ \mathrm{mA}\end{aligned}$$

Apply Nodal analysis at node 'a':

$$\begin{aligned}\frac{\mathbf{V}_a(\omega)}{\mathbf{Z}_{\mathrm{C}}} + \frac{\mathbf{V}_a(\omega)}{\mathbf{Z}_{\mathrm{L}} + R} &= \mathbf{I}(\omega)\\ \mathbf{V}_a(\omega) &= \frac{\mathbf{I}(\omega)}{\left(\frac{1}{\mathbf{Z}_{\mathrm{C}}} + \frac{1}{\mathbf{Z}_{\mathrm{L}} + R}\right)}\\ &= \frac{\mathbf{I}(\omega)\left(\mathbf{Z}_{\mathrm{C}}\right)\left(\mathbf{Z}_{\mathrm{L}} + R\right)}{\mathbf{Z}_{\mathrm{C}} + \mathbf{Z}_{\mathrm{L}} + R}\end{aligned}$$

Substitute the values,

$$\mathbf{V}_a(\omega) = \frac{(180\text{ mA})(-j5.55\ \Omega)(j150\ \Omega + 60\ \Omega)}{-j5.55\ \Omega + j150\ \Omega + 60\ \Omega}$$
$$= \frac{150 - j60\text{ A}\Omega^2}{60\ \Omega + j144.5\ \Omega}$$

The voltage division rule between the inductor and the resistor gives the voltage $\mathbf{V}(\omega)$ as:

$$\mathbf{V}(\omega) = \left(\frac{R}{R + \mathbf{Z}_{\text{L}}}\right)\mathbf{V}_a(\omega)$$

Substitute the values,

$$\mathbf{V}(\omega) = \left(\frac{60\ \Omega}{60\ \Omega + j150\ \Omega}\right)\left(\frac{150 - j60\text{ A}\Omega^2}{60\ \Omega + j144.5\ \Omega}\right)$$
$$= \frac{9000 - j3600\text{ A}\Omega^3}{-18075 + j17670\ \Omega^2}$$
$$= \frac{1800 - j720\text{ A}\Omega^3}{-3615 + j3534\ \Omega^2}$$
$$= -0.3542 - j0.1471\text{V}$$

The phase angle of the voltage $\mathbf{V}(\omega)$ is:

$$\theta = \tan^{-1}\left(\frac{-0.1471}{-0.3542}\right)$$
$$= -157.45°$$

The amplitude of voltage $\mathbf{V}(\omega)$ is:

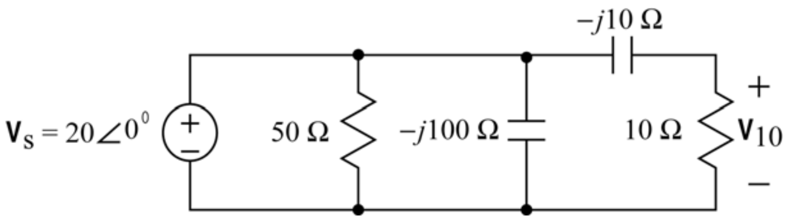
$$V = \sqrt{(0.3542\text{ V})^2 + (0.1471\text{ V})^2}$$
$$= 0.3835\text{ V}$$

The voltage across the resistor is:

$$v(t) = V\cos(6000t + \theta)$$
$$= 0.3835\cos(6000t - 157.45°)$$

Therefore, the voltage across the resistor is $\boxed{0.3835\cos(6000t - 157.45°)}$.

P 10.5-12
Solution:

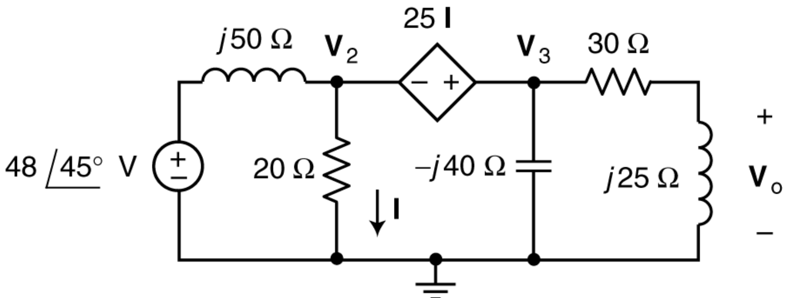


$$\mathbf{V}_{10} = \mathbf{V}_s\left(\frac{10}{10 - j10}\right)$$
$$= 20\angle 0°\left(\frac{10}{10\sqrt{2}\angle -45°}\right)$$
$$= 10\sqrt{2}\angle 45°$$

$$v_{10}(t) = 10\sqrt{2}\cos(100t + 45°)\ \text{V}$$

P10.6-1
Solution:

Represent the circuit in the frequency domain as



The node voltages are $48\angle 45° = \mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{V}_3$ and $\mathbf{V}_o$. Express the dependent source voltage in terms of the node voltages:

$$\mathbf{V}_3 - \mathbf{V}_2 = 25\mathbf{I} = 25\left(\frac{\mathbf{V}_2}{20}\right) \quad \Rightarrow \quad \mathbf{V}_3 = 2.25\,\mathbf{V}_2$$

Apply KCL to the supernode corresponding to the CCVS to get

$$\frac{48\angle 45° - \mathbf{V}_2}{j50} = \frac{\mathbf{V}_2}{20} + \frac{\mathbf{V}_3 - \mathbf{V}_o}{30} + \frac{\mathbf{V}_3}{-j40}$$

$$\frac{48\angle 45°}{j50} = \frac{\mathbf{V}_2}{j50} + \frac{\mathbf{V}_2}{20} + \frac{\mathbf{V}_3 - \mathbf{V}_o}{30} + \frac{\mathbf{V}_3}{-j40}$$

$$\frac{48\angle 45°}{j50} = \left(\frac{1}{j50} + \frac{1}{20}\right)\mathbf{V}_2 + \left(\frac{1}{30} + \frac{1}{-j40}\right)\mathbf{V}_3 - \frac{1}{30}\mathbf{V}_o$$

$$\frac{48\angle 45^\circ}{j50} = \left(\frac{1}{j50} + \frac{1}{20}\right)\mathbf{V}_2 + \left(\frac{1}{30} + \frac{1}{-j40}\right)2.25\mathbf{V}_2 - \frac{1}{30}\mathbf{V}_o$$

$$\frac{48\angle 45^\circ}{j50} = \left(\frac{1}{j50} + \frac{1}{20} + \frac{2.25}{30} + \frac{2.25}{-j40}\right)\mathbf{V}_2 - \frac{1}{30}\mathbf{V}_o$$

Apply KCL at the right node of the 30 Ω resistor to get

$$\frac{\mathbf{V}_3 - \mathbf{V}_o}{30} = \frac{\mathbf{V}_o}{j25} \quad \Rightarrow \quad 0 = \left(-\frac{1}{30}\right)2.25\mathbf{V}_2 + \left(\frac{1}{30} + \frac{1}{j25}\right)\mathbf{V}_o$$

In matrix form

$$\begin{bmatrix} \frac{1}{j50} + \frac{1}{20} + \frac{2.25}{30} + \frac{2.25}{-j40} & -\frac{1}{30} \\ -\frac{2.25}{30} & \frac{1}{30} + \frac{1}{j25} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} \frac{48\angle 45^\circ}{j50} \\ 0 \end{bmatrix}$$

Solving, perhaps using MATLAB,

$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} 10.18\angle -44.6^\circ \\ 14.67\angle 5.6^\circ \end{bmatrix} \text{ V}$$

MATLAB

```
>> A=[1/(j*50)+1/20+2.25/30+2.25/(-j*40) -1/30;-2.25/30 1/30+1/(j*25)]
A =
   0.1250 + 0.0363i  -0.0333 + 0.0000i
  -0.0750 + 0.0000i   0.0333 - 0.0400i
>> V=inv(A)*[48*exp(j*45*pi/180)/(j*50);0]
V =
   7.2496 - 7.1526i
  14.5998 + 1.4265i
>> abs(V)
ans =
   10.1841
   14.6694
>> angle(V)*180/pi
ans =
  -44.6139
    5.5805
```

P10.7-3
Solution:

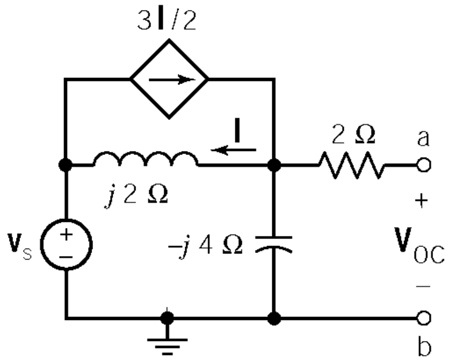
First, determine $\mathbf{V}_{oc}$:

The node equation is:

$$\frac{\mathbf{V}_{oc}}{-j4}+\frac{\mathbf{V}_{oc}-(6+j8)}{j2}-\frac{3}{2}\left(\frac{\mathbf{V}_{oc}-(6+j8)}{j2}\right)=0$$

$$\left(-\frac{1}{j4}+\frac{1}{j2}-\frac{3}{j4}\right)\mathbf{V}_{oc}=\frac{6+j8}{j2}-\frac{3}{2}\left(\frac{6+j8}{j2}\right)$$

$$\mathbf{V}_{oc}=3+j4=5\angle 53.1°\ \text{V}$$



$$\mathbf{V}_s=10\angle 53°=6+j8\ \text{V}$$

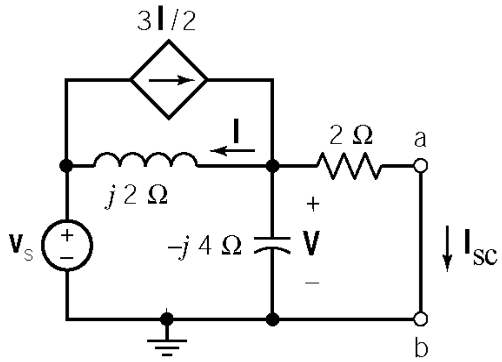
Next, determine $\mathbf{I}_{sc}$:

The node equation is:

$$\frac{\mathbf{V}}{2}+\frac{\mathbf{V}}{-j4}+\frac{\mathbf{V}-(6+j8)}{j2}-\frac{3}{2}\left[\frac{\mathbf{V}-(6+j8)}{j2}\right]=0$$

$$\mathbf{V}=\frac{3+j4}{1-j}$$

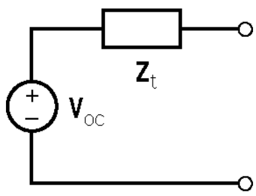
$$\mathbf{I}_{sc}=\frac{\mathbf{V}}{2}=\frac{3+j4}{2-j2}$$



$$\mathbf{V}_s=10\angle 53°=6+j8\ \text{V}$$

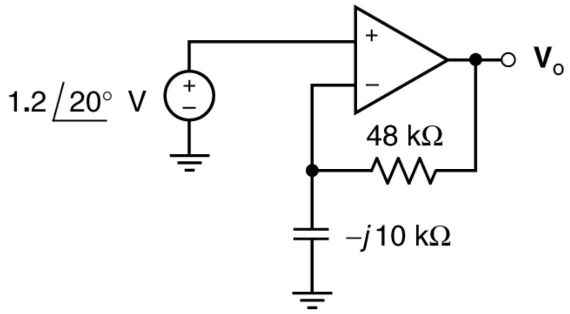
The Thevenin impedance is $\mathbf{Z}_T=\dfrac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}}=(3+j4)\left(\dfrac{2-j2}{3+j4}\right)=2-j2\ \Omega$

The Thevenin equivalent is



(checked: LNAP 7/18/04)

P10.10-1
Solution:
Represent the circuit in the frequency domain as



Recognizing this circuit as a noninverting amplifier, we can write

$$\mathbf{V}_0 = \left(1 + \frac{48}{-j10}\right)(1.2\angle 20°) = (1 + j4.8)(1.2\angle 20°) = 5.88\angle 98° \text{ V}$$

In the time domain $\quad v_o(t) = 5.88\cos(400t + 98°) \text{ V}$

(Checked using LNAPAC 3/15/12)