P 14.4-1
Solution

$$F(s)=\frac{s+5}{s^3+3s^2+6s+4}=\frac{s+5}{(s+1)\left[(s+1)^2+3\right]}=\frac{A}{s+1}+\frac{Bs+C}{s^2+2s+4}$$

where

$$A=\left.\frac{s+5}{(s+1)^2+3}\right|_{s=-1}=\frac{4}{3}$$

Then

$$\frac{(s+5)}{(s+1)(s^2+2s+4)}=\frac{\frac{4}{3}}{s+1}+\frac{Bs+C}{s^2+2s+4}\Rightarrow(s+5)=\left(\frac{4}{3}+B\right)s^2+\left(\frac{8}{3}+B+C\right)+\frac{16}{3}+C$$

Equating coefficient yields

$$s^2:\quad 0=\frac{4}{3}+B\quad\Rightarrow\quad B=-\frac{4}{3}$$

$$s\ :\quad 1=\frac{8}{3}-\frac{4}{3}+C\quad\Rightarrow\quad C=-\frac{1}{3}$$

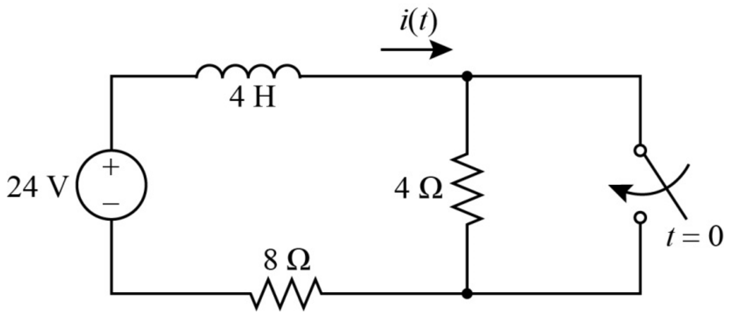
Then

$$F(s)=\frac{\frac{4}{3}}{s+1}+\frac{-\frac{4}{3}s+\frac{1}{3}}{(s+1)^2+3}=\frac{\frac{4}{3}}{s+1}+\frac{-\frac{4}{3}(s+1)}{(s+1)^2+3}+\frac{\frac{1}{\sqrt{3}}\sqrt{3}}{(s+1)^2+3}$$

Taking the inverse Laplace transform yields

$$f(t)=\frac{4}{3}e^{-t}-\frac{4}{3}e^{-t}\cos\sqrt{3}t+\frac{1}{\sqrt{3}}e^{-t}\sin\sqrt{3}\,t$$

P14.6-1
Solution:
In order to find the inductor current $i(t)$ proceed as follows:



When the switch is closed for the circuit shown in the figure above, the Kirchhoff's Voltage Law (KVL) is applied to obtain the inductor current $i(t)$ as follows:

$$-24+4\frac{di(t)}{dt}+8i(t)=0$$

$$i(0)=\frac{24}{4+8}=2$$

Apply Laplace Transform:

$$4sI(s)\text{-}4i(0)+8I(s)=\frac{24}{s}$$

$$I(s)[4s+8]=\frac{24}{s}+8$$

$$i(s)=-\frac{24}{s(4s+8)}+\frac{8}{4s+8}$$

$$=\frac{6}{s(s+2)}+\frac{2}{s+2}=\frac{3}{s}-\frac{3}{s+2}+\frac{2}{s+2}=\frac{3}{s}-\frac{1}{s+2}$$

Now apply the Inverse Laplace transform to obtain $i(t)$ as follows:

$$i(t)=3-3e^{-2t}+2e^{-2t}$$

$$=3-e^{-2t}$$

Hence, inductor current $i(t)$ is $\boxed{3-e^{-2t}}$

P 14.7-14 Determine the current $i_L(t)$ for $t \geq 0$ for the circuit of Figure P 14.7-14.

Hint: $v_C(0) = 8$ V and $i_L(0) = 1$ A

Answer: $i_L(t) = \left(e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t\right)u(t)$ A

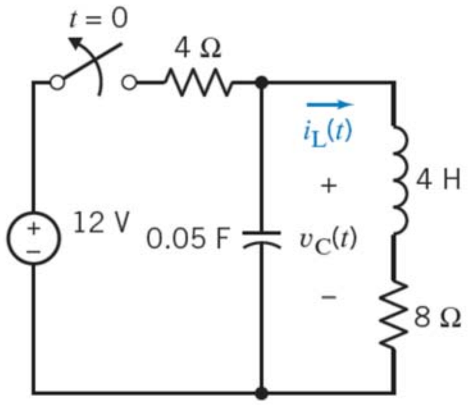
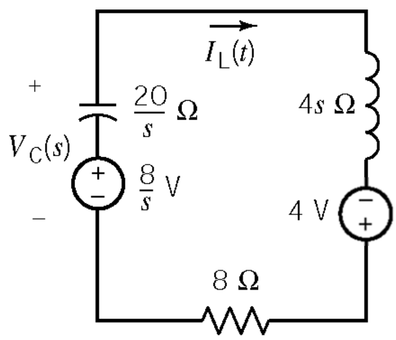


Figure P 14.7-14

P 14.7-14

Solution:



KVL:

$$\frac{8}{s} + 4 = \left(\frac{20}{s} + 8 + 4s\right) I_L(s)$$

so

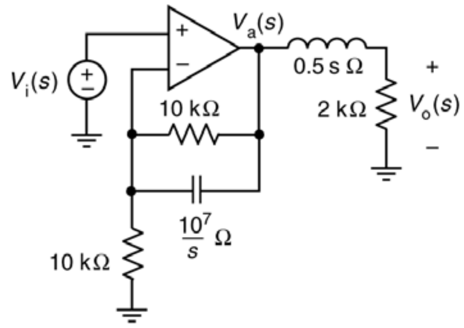
$$I_L(s) = \frac{2+s}{s^2+2s+5} = \frac{(s+1)+1}{(s+1)^2+4}$$

Taking the inverse Laplace transform:

$$i_L(t) = \left(e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t\right)u(t) \text{ A}$$

P14.8-7

Solution: First, determine the transfer function from the circuit. To do so, represent the circuit in the s-domain as shown.



Recognizing the noninverting amplifier we

$$V_a(s) = \left(1 + \frac{\frac{10^7}{s} \| 10^4}{10^4}\right) V_i(s) = \left(1 + \frac{\frac{10^7}{s+10^3}}{10^4}\right) V_i(s)$$

$$= \left(\frac{s+2000}{s+1000}\right) V_i(s)$$

(Alternately, this equation can be obtained by applying KCL at the inverting input node of the op amp.)

Use voltage division to write

$$V_o(s) = \frac{2000}{2000 + 0.5\,s} V_a(s) = \frac{4000}{s+2000} V_a(s)$$

The transfer function is

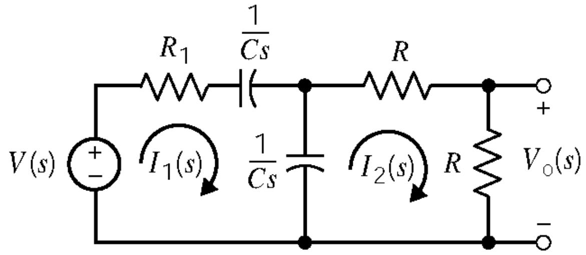
$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{4000(s+2000)}{(s+1000)(s+4000)}$$

The step response is

$$v_o(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathcal{L}^{-1}\left[\frac{4000(s+2000)}{s(s+1000)(s+4000)}\right] = \mathcal{L}^{-1}\left[\frac{2}{s} + \frac{-4/3}{s+1000} + \frac{-2/3}{s+4000}\right].$$

$$v_o(t) = \left[2 - \left(\frac{4}{3}e^{-1000t} + \frac{2}{3}e^{-4000t}\right)\right] u(t) \text{ V}$$

P 14.8-9
Solution:



Mesh equations:

$$V(s) = \left(R_1 + \frac{1}{Cs} + \frac{1}{Cs}\right) I_1(s) - \frac{1}{Cs} I_2(s)$$

$$0 = \left(R + R + \frac{1}{Cs}\right) I_2(s) - \frac{1}{Cs} I_1(s)$$

Solving for $I_2(s)$:
$$I_2(s) = \frac{V(s)\left(\frac{1}{Cs}\right)}{\left(R_1+\frac{2}{Cs}\right)\left(2R+\frac{1}{Cs}\right) - \frac{1}{(Cs)^2}}$$

Then $V_o(s) = R I_2(s)$ gives

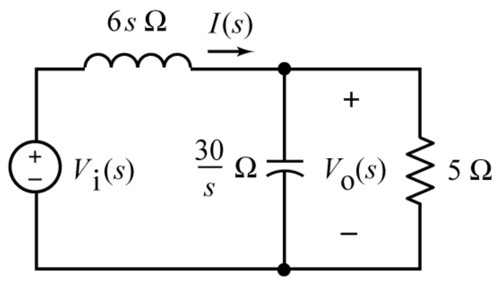
$$H(s) = \frac{V_0(s)}{V(s)} = \frac{RCs}{[R_1Cs+2][2RCs+1]-1} = \frac{s}{2R_1C\left[s^2 + \frac{4RC+R_1C}{2RR_1C^2}s + \frac{1}{\left(2RR_1C^2\right)^2}\right]}$$

P14.9-1

Solution:
To solve this problem using convolution, we first represent the input is by the function

$$v_i(t) = \begin{cases} 0 & t \le 2 \\ 10t-20 & 2 \le t \le 3 \\ -7.5t+32.5 & 3 \le t \le 5 \\ 5t-30 & 5 \le t \le 6 \\ 0 & 6 \le t \end{cases}$$

Next, we obtain the impulse response. To do so, assume that the initial conditions are zero and represent the circuit in the s-domain as



Using voltage division and equivalent impedance, the transfer function is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{30}{s}\,\|\,5}{6s+\frac{30}{s}\,\|\,5} = \frac{\frac{30}{s+6}}{6s+\frac{30}{s+6}} = \frac{5}{s^2+6s+5} = \frac{1.25}{s+1} - \frac{1.25}{s+5}$$

The impulse response is

$$h(t) = \mathcal{L}^{-1}\left[H(s)\right] = \mathcal{L}^{-1}\left[\frac{1.25}{s+1} - \frac{1.25}{s+5}\right] = \left(1.25e^{-t} - 1.25e^{-5t}\right)u(t)$$

Edit the MATLAB script from Example 14.9.1 to obtain

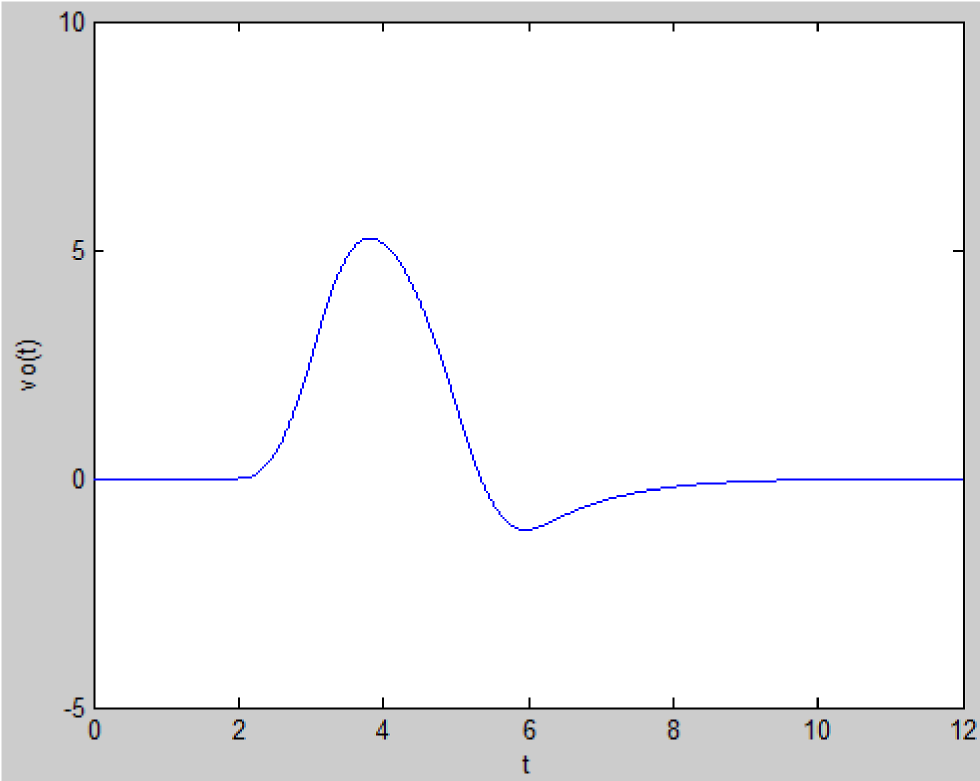
```
% P14_9_1.m - plots the output for Problem
14.9-1
% ---------------------------------------------
------
%  Obtain a list of equally spaced instants of
time
% ---------------------------------------------
------
t0 = 0; % begin
tf = 12;  % end
N = 5000;  % number of points plotted
dt = (tf-t0)/N; % increment
t = t0:dt:tf; % time in seconds

% ---------------------------------------------
------
% Obtain the input x(t) and the impulse
response h(t)
% ---------------------------------------------
------
for k = 1 : length(t)
    if t(k) < 2
        x(k) = 0;
    elseif t(k) < 3
        x(k) = -20 + 10*t(k); %
    elseif t(k) < 5
        x(k) = +32.5 - 7.5*t(k); %
    elseif t(k) < 6
        x(k) = -30 + 5*t(k); %
    else
        x(k) = 0;
    end
end
x=x*dt;
h=1.25*exp(-t)-1.25*exp(-5*t);

% ---------------------------------------------
------
%            Perform the convolution
% ---------------------------------------------
------
y=conv(x,h);

% ---------------------------------------------
------
%            Plot the output y(t)
% ---------------------------------------------
------
plot(t,y(1:length(t)))
axis([t0, tf, -5, 10])
xlabel('t')
ylabel('y(t)')
```

Running this script produces the required plot of the output voltage:



10
5
0
-5
vo(t)
0
2
4
6
8
10
12
t