MATLAB Problem

EFS.m

```
function [C0, Cn] = EFS(N, T)
%EFS Exponential Fourier Series
% returns the coefficients of the exponential Fourier
% Series of a periofic function described in the
% MATLAB function named my_periodic_function.m
%
% N = the number of harmonic frequencies
% T = the period of the periodic function
%
% C0=average value
% Cn(1)=C1, Cn(2)=C2, ..., Cn(N)=CN

% ---------------------------------------------------
%  Obtain a list of equally spaced instants of time
% ---------------------------------------------------
n=2*N;
t=linspace(0,T,n+1);
t(end)=[];
% ---------------------------------------------------
%  Obtain values of f(t) at those instants of time
% ---------------------------------------------------
f=my_squarewave(t,T);
```

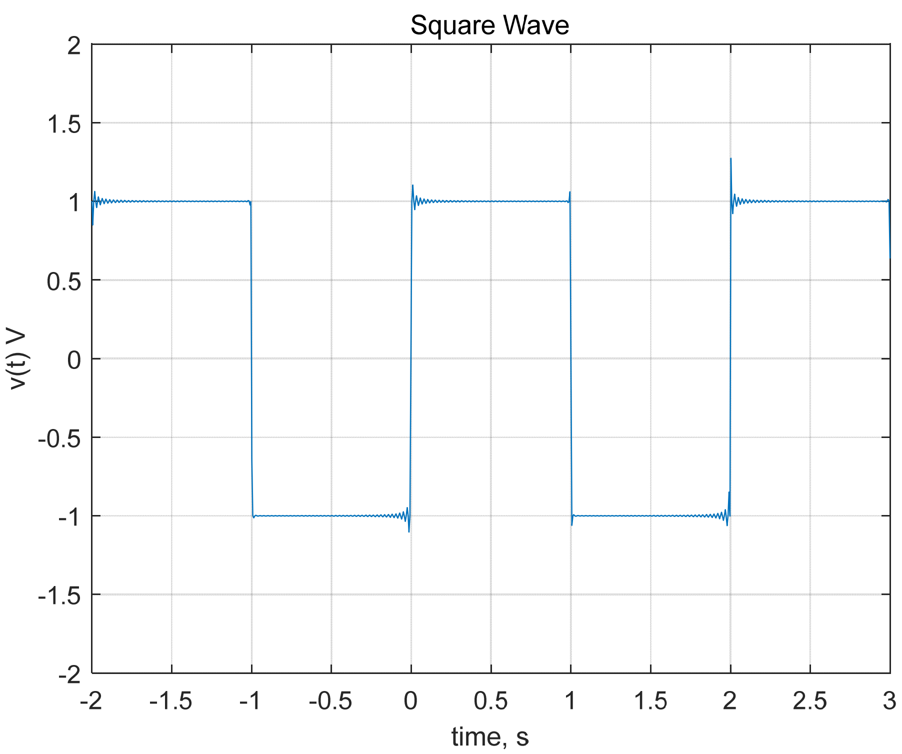
my_squarewave.m

```
function f = my_squarewave(t, T)
% % sawtooth with amplitude A and period T
% A=8;
% f=A*rem(t,T)/T;
% f(f==0 |f==A) = A/2;

% squarewave
A=1;
for k=1:length(t)
    if (t(k)<T/2) f(k)=A;
    elseif (t(k)>T/2)  f(k)=-A;
    else f(k)= 0;
    end
end
```

testEFS.m

```
% testEFS.m
% --------------------------------------------------
%  Obtain a list of equally spaced instants of time
% --------------------------------------------------
T=2;                % period
w0=2*pi/T;          % fundamental frequency, rad/s
t0=-T;              % initial time
tf=1.5*T;           % final time
dt=tf/512;          % time increment
t=-T:dt:tf;         % time, s
% --------------------------------------------------
%  Call EFS to get exponential Fourier coefficients
% --------------------------------------------------
N=256; %Number of harmonic frequencies
[C0, Cn] = EFS(N,T);
% --------------------------------------------------
%  Approximate the function by its Fourier series
% --------------------------------------------------
v = C0*ones(size(t));  % initialize v(t) as vector
for n=1:N
   v = v + Cn(n)*exp(j*n*w0*t) + Cn(n)'*exp(-j*n*w0*t);
end
% --------------------------------------------------
%               Plot the Fourier series
% --------------------------------------------------
plot(t, v)
axis([t0 tf -2 2])
```



P 15.5-2
Solution:

$$\mathbf{C}_n = \frac{1}{2T}\int_0^{2T}\left(\frac{A}{2T}t\right)e^{-j\frac{2\pi}{T}nt}\,dt$$
$$= \frac{A}{T^2}\int_0^{2T} te^{-j(\pi/T)nt}\,dt$$

Recall the formula for integrating by parts: $\int_{t_1}^{t_2} u\,dv = u\,v\big|_{t_1}^{t_2} - \int_{t_1}^{t_2} v\,du$. Take $u = t$ and $dv = e^{-j\frac{\pi}{T}nt}dt$. When $n \neq 0$, we get

$$\mathbf{C}_n = \frac{A}{4T^2}\left(\left.\frac{te^{-j(\pi/T)nt}}{-j(\pi/T)n}\right|_0^T + \frac{1}{j(\pi/T)n}\int_0^{2T} e^{-j(\pi/T)nt}dt\right)$$
$$= \frac{A}{4T^2}\left(\left.\frac{te^{-j(\pi/T)nt}}{-j(\pi/T)n} + \frac{e^{-j(\pi/T)nt}}{(\pi/T)^2 n^2}\right|_0^{2T}\right)$$
$$= \frac{A}{4T^2}\left(\frac{2Te^{-j2\pi n}}{-j2\pi n} + \frac{e^{-j2\pi n}-1}{(\pi/T)^2 n^2}\right)$$
$$= j\frac{A}{2\pi n}$$

Now for $n = 0$ we have

$$C_0 = \frac{1}{2T}\int_0^{2T}\frac{A}{2T}t\,dt = \frac{A}{2}$$

Finally,

$$f(t) = \frac{A}{2} + j\frac{A}{2\pi}\sum_{\substack{n=-\infty \\ n\neq 0}}^{n=\infty}\frac{1}{n}e^{jn\frac{\pi}{T}t}$$

P 15.9-6
Solution:

$$F(\omega)=\int_{-2}^{2}e^{-j\omega t}dt-\int_{-1}^{1}e^{-j\omega t}dt=\left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-2}^{2}-\left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-1}^{1}=\frac{1}{j\omega}\left(e^{j2\omega}-e^{-j2\omega}\right)-\frac{1}{j\omega}\left(e^{j\omega}-e^{-j\omega}\right)$$

$$=\frac{2}{\omega}\left(\sin 2\omega-\sin\omega\right)$$

P 16.3-4
Solution:
Use Table 16-3.2 to obtain the transfer function of a fourth-order Butterworth high-pass filter having a cutoff frequency equal to 1 rad/s and a dc gain equal to 5.

$$H_n(s)=\frac{5s^4}{\left(s^2+0.765s+1\right)\left(s^2+1.848s+1\right)}$$

Frequency scaling can be used to adjust the cutoff frequency 400 hertz = 2513 rad/s:

$$H_{\rm H}(s)=\frac{5\cdot\left(\frac{s}{2513}\right)^4}{\left(\left(\frac{s}{2513}\right)^2+0.765\left(\frac{s}{2513}\right)+1\right)\left(\left(\frac{s}{2513}\right)^2+1.848\left(\frac{s}{2513}\right)+1\right)}=\frac{5\cdot s^4}{\left(s^2+1922.4s+2513^2\right)\left(s^2+4644.025s+2513^2\right)}$$

P 16.4-1
Solution
The transfer function is

$$H(s)=\frac{V_{\rm o}(s)}{V_{\rm s}(s)}=\frac{\dfrac{sL\times\frac{1}{Cs}}{sL+\frac{1}{Cs}}}{\dfrac{sL\times\frac{1}{Cs}}{sL+\frac{1}{Cs}}+R}=\frac{\dfrac{sL}{s^2LC+1}}{\dfrac{sL}{s^2LC+1}+R}=\frac{sL}{s^2LCR+sL+R}=\frac{\dfrac{s}{RC}}{s^2+\dfrac{s}{RC}+\dfrac{1}{LC}}$$

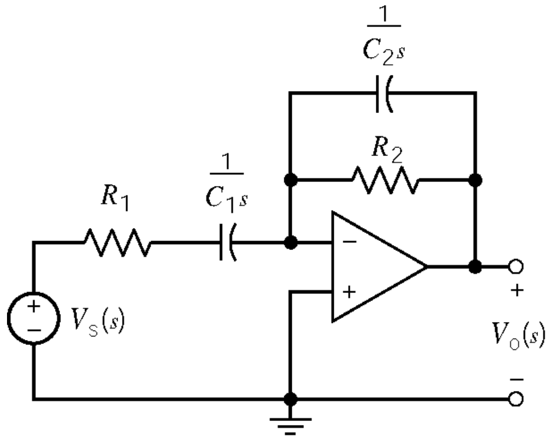
so

$$K=1\ ,\ \omega_0^2=\frac{1}{LC}\quad\text{and}\quad\frac{1}{RC}=\frac{\omega_0}{Q}\Rightarrow Q=RC\omega_0=R\sqrt{\frac{C}{L}}$$

Pick $C=1\ \mu\text{F}$. Then $L=\dfrac{1}{C\omega_0^2}=0.4$ H and $R=Q\sqrt{\dfrac{L}{C}}=632\ \Omega$

P 16.4-11

Solution:



$$H(s)=\frac{V_0(s)}{V_S(s)}=-\frac{R_2 \left\| \frac{1}{C_2 s} \right.}{R_1+\frac{1}{C_1 s}}=-\frac{\frac{R_2}{R_2 C_2 s+1}}{\frac{R_1 C_1 s+1}{C_1 s}}=\frac{-\frac{1}{R_1 C_2} s}{s^2+\left(\frac{1}{R_1 C_1}+\frac{1}{R_2 C_2}\right) s+\frac{1}{R_1 R_2 C_1 C_2}}$$

Substituting the element values $R_1 = 200\ \Omega$, $R_2 = 200\ \text{k}\Omega$, $C_1 = 0.4\ \mu\text{F}$ and $C_2 = 50\ \text{pF}$ we determine the center frequency and bandwidth of the filter to be

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} = 35.4\ \text{k rad/sec} = 2\pi(5.63\ \text{kHz})$$

$$BW = \frac{\omega_0}{Q} = \frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} = 112.5\ \text{k rad}/s = 2\pi(17.9\ \text{kHz})$$