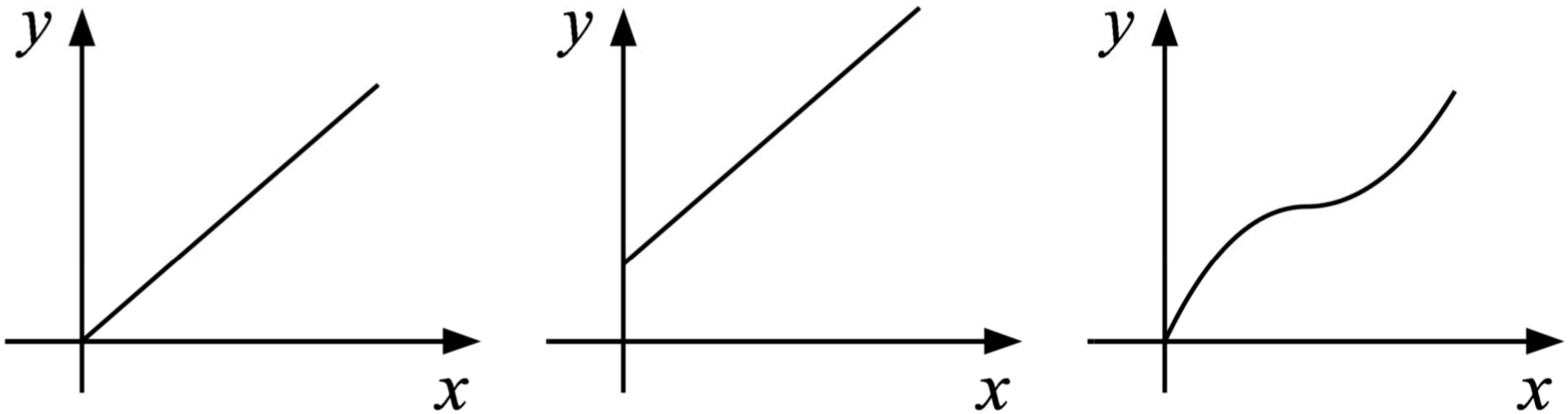
Introduction to Electric Circuits

Review

Linear Time-invariant Circuits

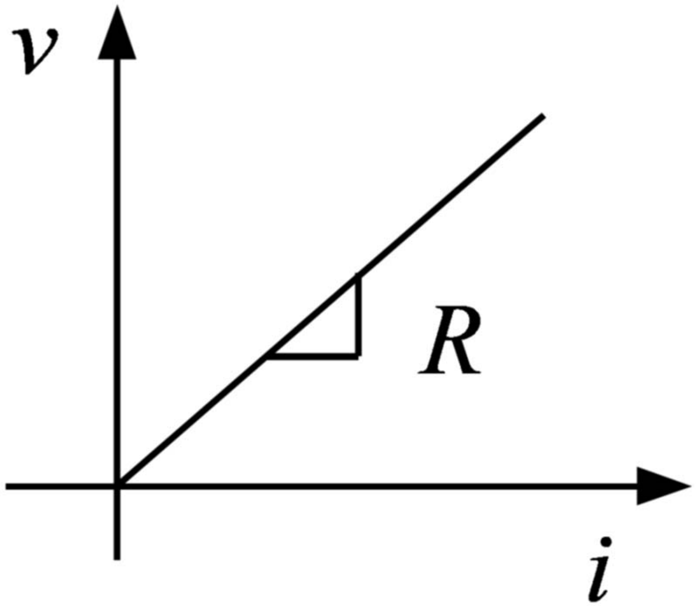
- Linear function, Nonlinear function

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$
$$f(Kx) = Kf(x)$$

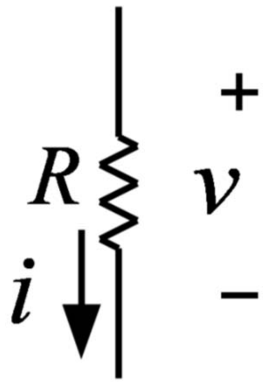


$y = f(x) = Kx$ $\quad$ $y = f(x) = Kx + b$ $\quad$ $y = f(x)$

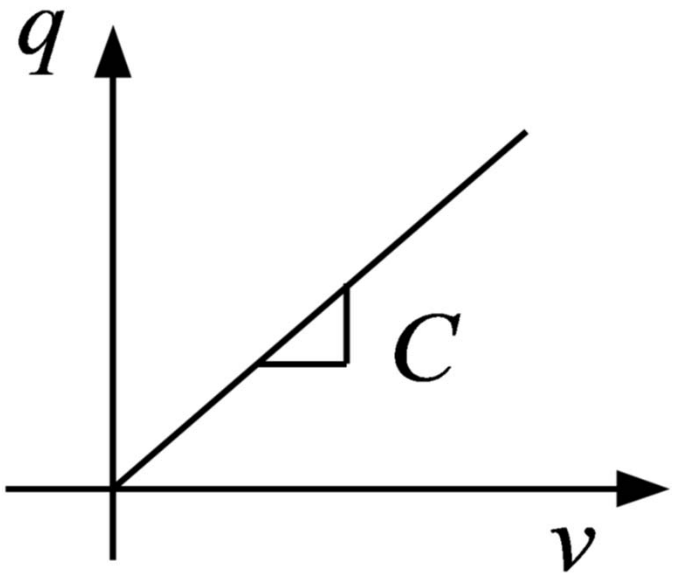
Linear Resistor



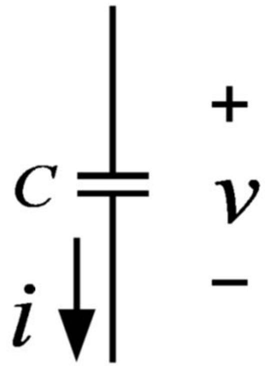
$$v = Ri$$

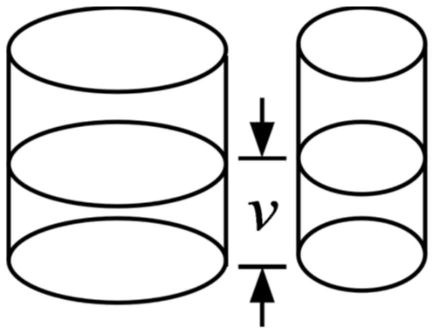


Linear Capacitor



$$q = Cv$$



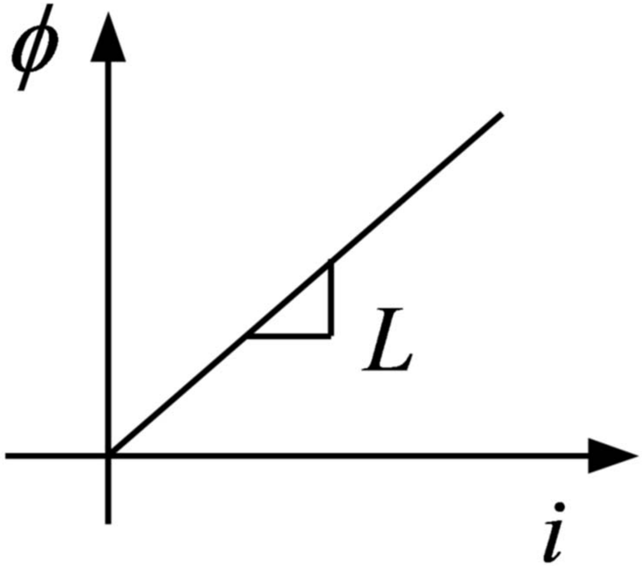


$$C_1 > C_2$$

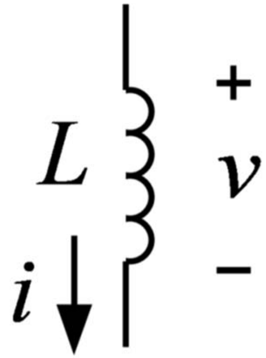
$$q_1 = C_1 v > q_2 = C_2 v$$

$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

Linear Inductor

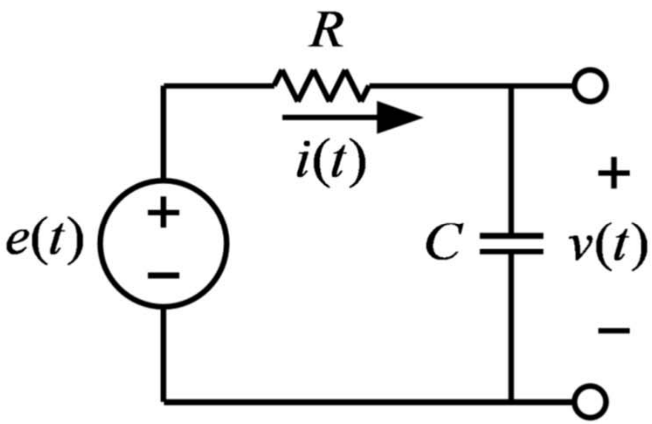


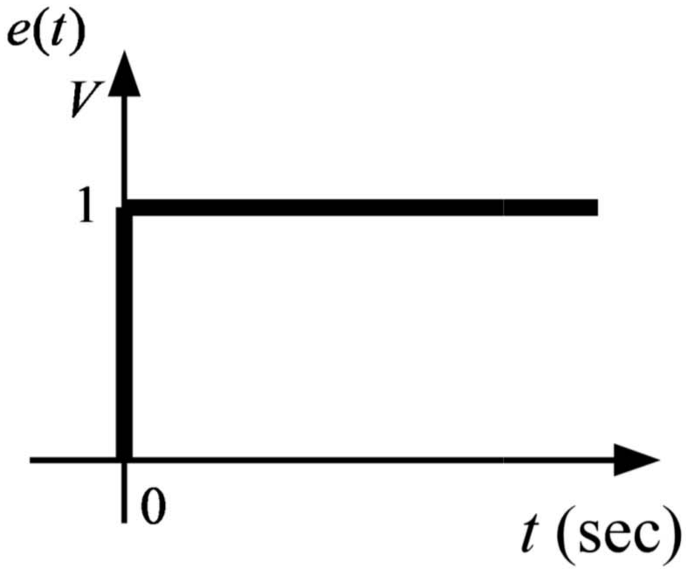
$$\phi = Li$$



$$v = \frac{d\phi}{dt} = L\frac{di}{dt}$$

RC Circuit





$KVL:$

$$Ri + v = e, \quad i = C\frac{dv}{dt}$$

$$RC\frac{dv}{dt} + v = e, \quad v(0) = 0$$

- Complete response = natural response + forced response
- Natural response(homogeneous solution):

$$RC\frac{dv}{dt}+v=0$$

$$RCs+1=0\Rightarrow s=-\frac{1}{RC}$$

$$v(t)=Ke^{-\frac{t}{RC}}$$

- Forced response(particular solution), steady state:

$$v=V_{ss}$$

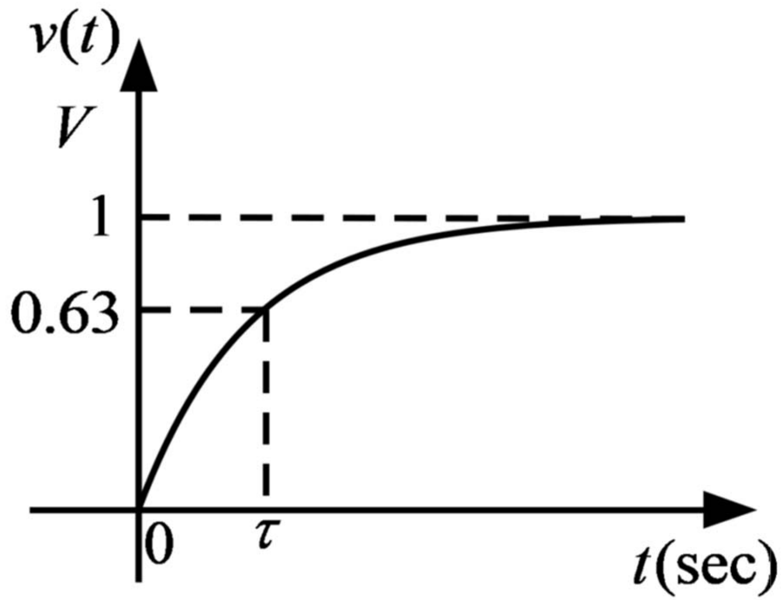
$$RC\frac{dV_{ss}}{dt}+V_{ss}=1\Rightarrow V_{ss}=1$$

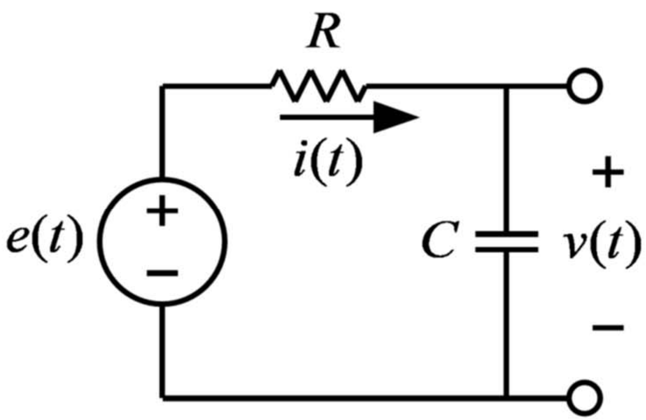
- Complete response = natural response + forced response

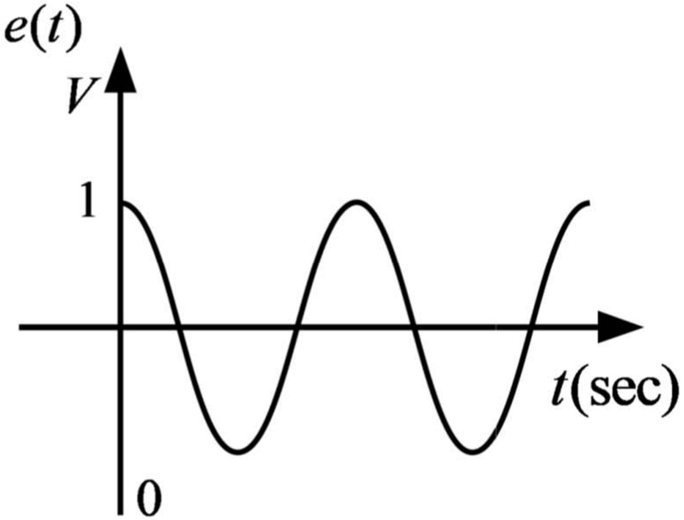
$$v(t) = Ke^{-\frac{t}{RC}} + 1$$

$$v(0) = K + 1 = 0 \Rightarrow K = -1$$

$$v(t) = -e^{-\frac{t}{RC}} + 1$$







$$KVL:$$

$$Ri + v = e = \cos \omega t, \quad i = C\frac{dv}{dt}$$

$$RC\frac{dv}{dt} + v = \cos \omega t, \quad v(0) = 1$$

- Forced response(particular solution), steady state:

$$v = A\cos\omega t + B\sin\omega t$$

$$RC\frac{d\left(A\cos\omega t + B\sin\omega t\right)}{dt} + \left(A\cos\omega t + B\sin\omega t\right) = \cos\omega t$$

$$-\omega RCA\sin\omega t + \omega RCB\cos\omega t + A\cos\omega t + B\sin\omega t = \cos\omega t$$

$$\left(\omega RCB + A\right)\cos\omega t + \left(B - \omega RCA\right)\sin\omega t = \cos\omega$$

$$\omega RCB + A = 1, B - \omega RCA = 0$$

$$B = \omega RCA, \omega RC\left(\omega RCA\right) + A = 1$$

$$A = \frac{1}{\left(\omega RC\right)^2 + 1}, B = \frac{\omega RC}{\left(\omega RC\right)^2 + 1}$$

- Forced response(particular solution), steady state:

$$v = A\cos\omega t + B\sin\omega t$$

$$= \frac{1}{(\omega RC)^2 + 1}\cos\omega t + \frac{\omega RC}{(\omega RC)^2 + 1}\sin\omega t$$

$$= \frac{\sqrt{(\omega RC)^2 + 1}}{(\omega RC)^2 + 1}\cos(\omega t - \tan^{-1}\omega RC)$$

$$= \frac{1}{\sqrt{(\omega RC)^2 + 1}}\cos(\omega t - \tan^{-1}\omega RC)$$

- Complete response

$$v(t) = Ke^{-\frac{t}{RC}} + \frac{1}{(\omega RC)^2 + 1}\cos\omega t + \frac{\omega RC}{(\omega RC)^2 + 1}\sin\omega t$$

$$= Ke^{-\frac{t}{RC}} + \frac{1}{\sqrt{(\omega RC)^2 + 1}}\cos(\omega t - \tan^{-1}\omega RC)$$

$$v(0) = K + \frac{1}{(\omega RC)^2 + 1} = 1, K = \frac{(\omega RC)^2}{(\omega RC)^2 + 1}$$

$$v(t) = \frac{(\omega RC)^2}{(\omega RC)^2 + 1}e^{-\frac{t}{RC}} + \frac{1}{(\omega RC)^2 + 1}\cos\omega t + \frac{\omega RC}{(\omega RC)^2 + 1}\sin\omega t$$

- Check

$$v(t) = \frac{(\omega RC)^2}{(\omega RC)^2+1} e^{-\frac{t}{RC}} + \frac{1}{(\omega RC)^2+1}\cos\omega t + \frac{\omega RC}{(\omega RC)^2+1}\sin\omega t$$

$$\frac{dv(t)}{dt} = -\frac{1}{RC}\frac{(\omega RC)^2}{(\omega RC)^2+1} e^{-\frac{t}{RC}} - \frac{\omega}{(\omega RC)^2+1}\sin\omega t + \frac{\omega^2 RC}{(RC\omega)^2+1}\cos\omega t$$

$$\left.\frac{dv(t)}{dt}\right|_{t=0} = -\frac{1}{RC}\frac{(\omega RC)^2}{(\omega RC)^2+1} + \frac{\omega^2 RC}{(\omega RC)^2+1} = 0$$