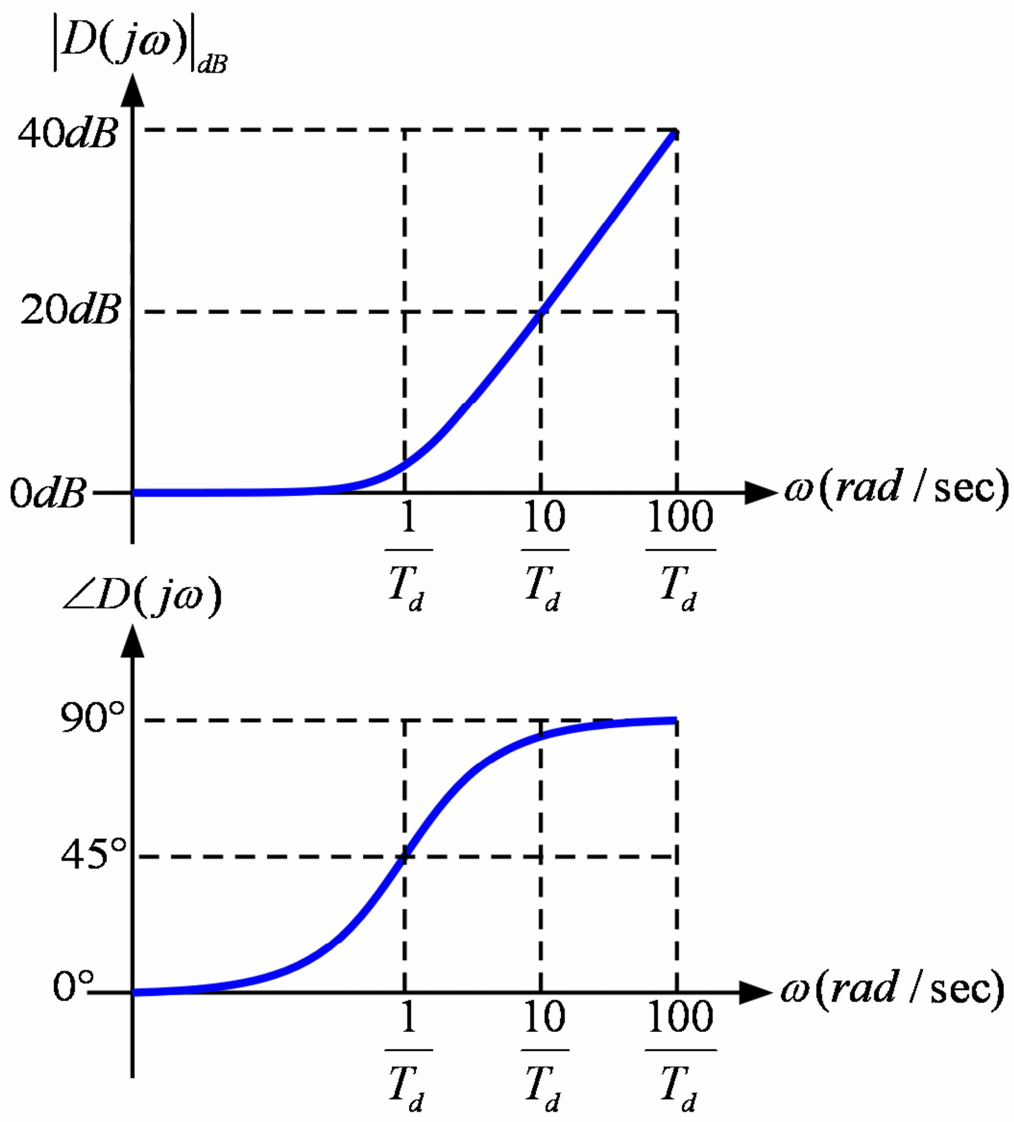
PD 제어기의 설계

$$D(s) = K\left(1 + T_d s\right)$$



예제: PD 제어기

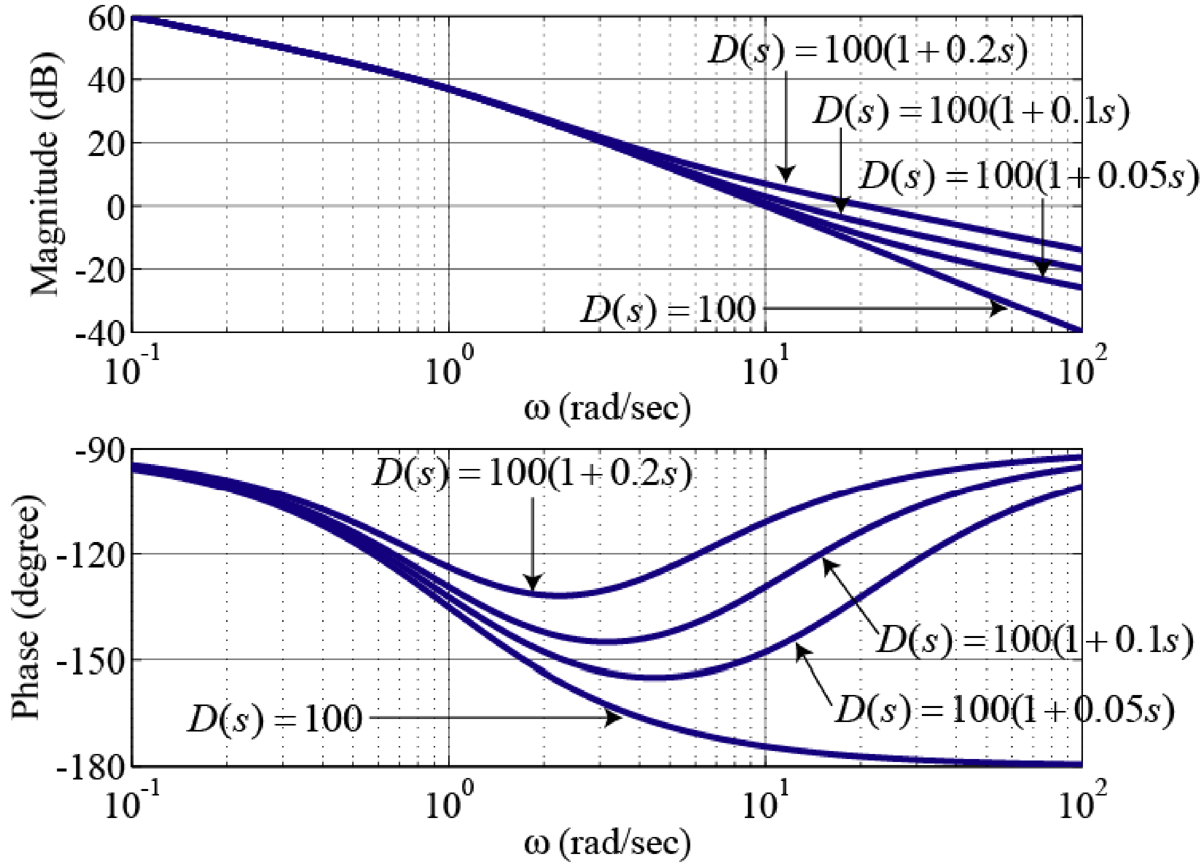
$$G(s) = \frac{1}{s(s+1)}$$

$$K_v = \lim_{s \to 0} s \frac{K}{s(s+1)} = K$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K}$$

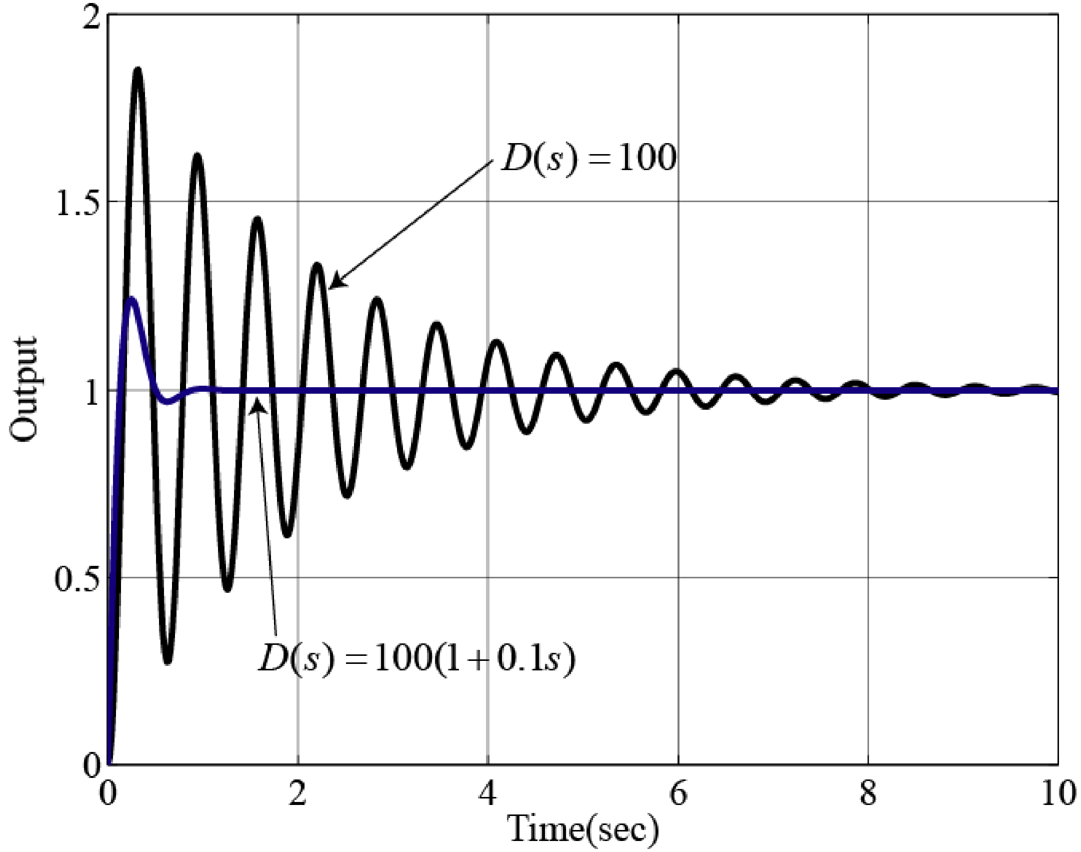
$$D(s) = 100(1 + 0.1s)$$

예제: PD 제어기



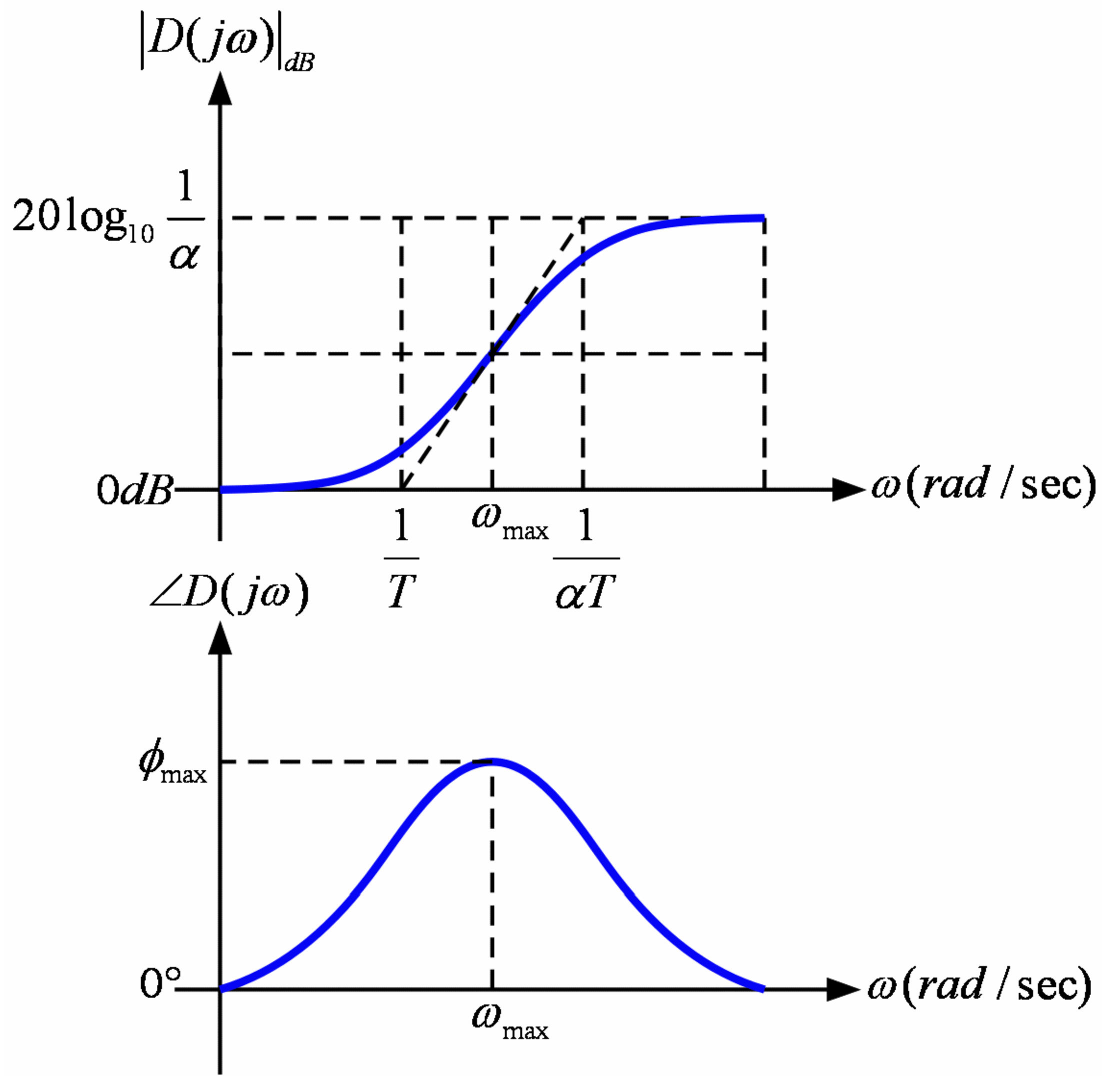
60
40
20
0
-20
-40
Magnitude (dB)
D(s) = 100(1 + 0.2s)
D(s) = 100(1 + 0.1s)
D(s) = 100(1 + 0.05s)
D(s) = 100
10^-1
10^0
10^1
10^2
ω (rad/sec)
-90
-120
-150
-180
Phase (degree)
D(s) = 100(1 + 0.2s)
D(s) = 100(1 + 0.1s)
D(s) = 100(1 + 0.05s)
D(s) = 100
10^-1
10^0
10^1
10^2
ω (rad/sec)

예제: PD 제어기



진상 제어기(Lead Compensator) 의 설계

$$D(s) = K\frac{Ts+1}{\alpha Ts+1}$$



진상 제어기의 설계

$$\phi = \angle\left(\frac{jT\omega+1}{j\alpha T\omega+1}\right) = \tan^{-1}(T\omega) - \tan^{-1}(\alpha T\omega)$$

$$\log_{10}\omega_{\max} = \frac{1}{2}\left(\log_{10}\frac{1}{T} + \log_{10}\frac{1}{\alpha T}\right) \qquad \omega_{\max} = \frac{1}{T\sqrt{\alpha}}$$

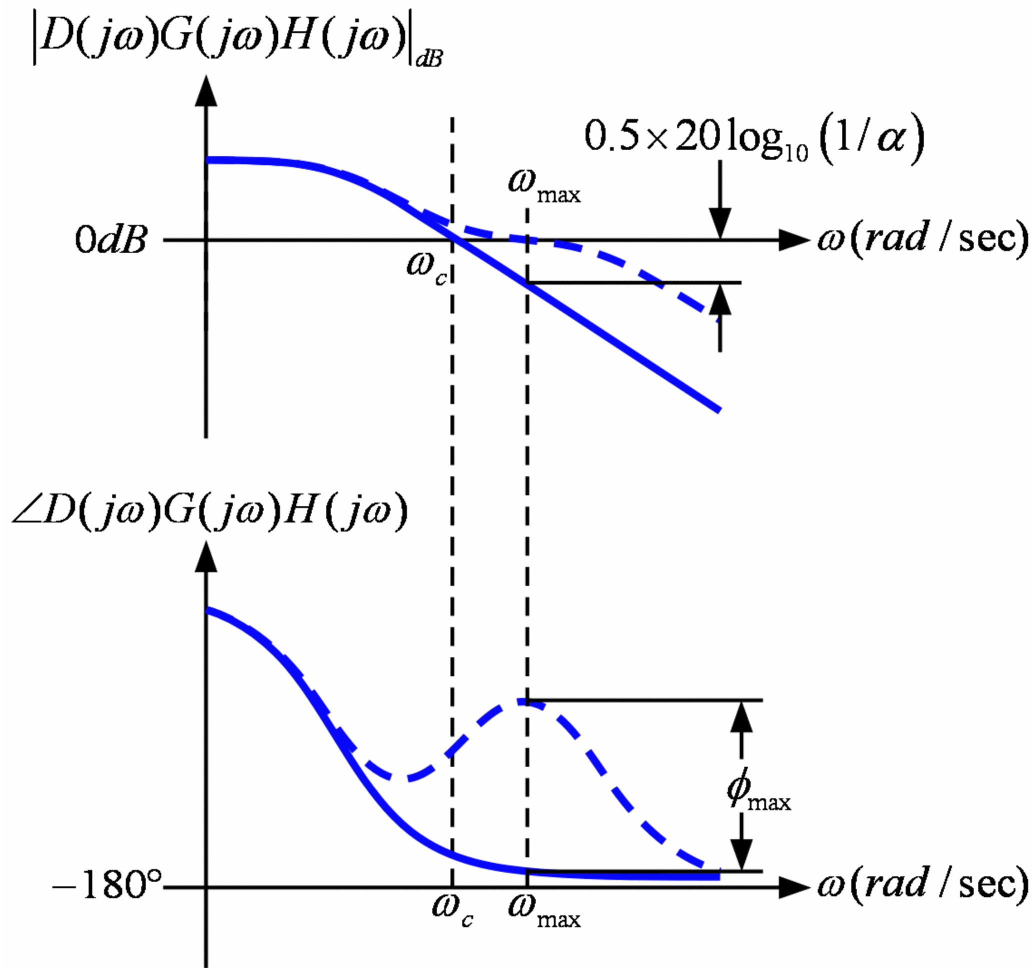
$$\phi_{\max} = \tan^{-1}\frac{1}{\sqrt{\alpha}} - \tan^{-1}\sqrt{\alpha}$$

$$\tan\phi_{\max} = \frac{1-\alpha}{2\sqrt{\alpha}} \qquad \sin\phi_{\max} = \frac{1-\alpha}{1+\alpha} \qquad \alpha = \frac{1-\sin\phi_{\max}}{1+\sin\phi_{\max}}$$

진상 제어기의 설계

$$20\log_{10}\left|KG(j\omega_{\max})H(j\omega_{\max})\right| = -0.5\times 20\log_{10}\frac{1}{\alpha}$$

$$T = \frac{1}{\omega_{\max}\sqrt{\alpha}}$$



예제: 진상 제어기

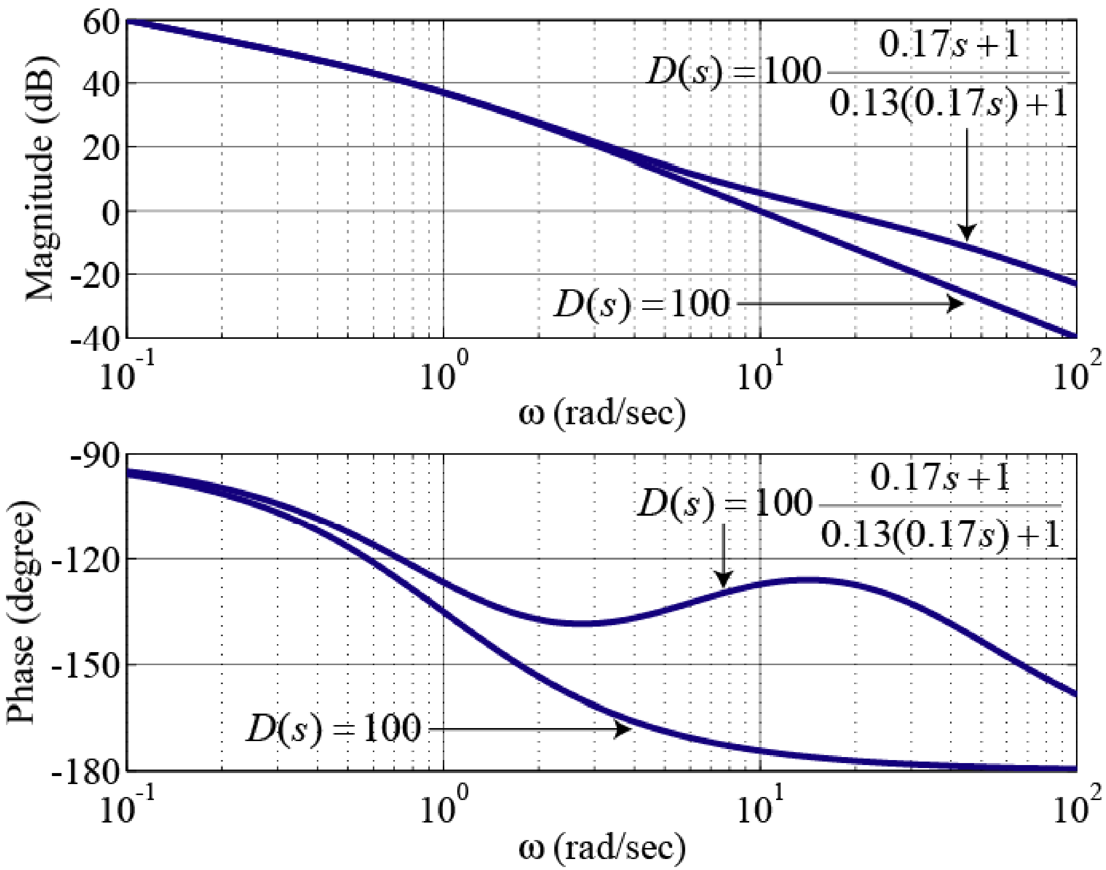
$$\phi_{\max} = 50^\circ \qquad \alpha = 0.13$$

$$0.5 \times 20\log_{10}\left(1/\alpha\right) = 9dB$$

$$T = \frac{1}{\omega_{\max}\sqrt{\alpha}} = \frac{1}{16.7\sqrt{0.13}} = 0.17$$

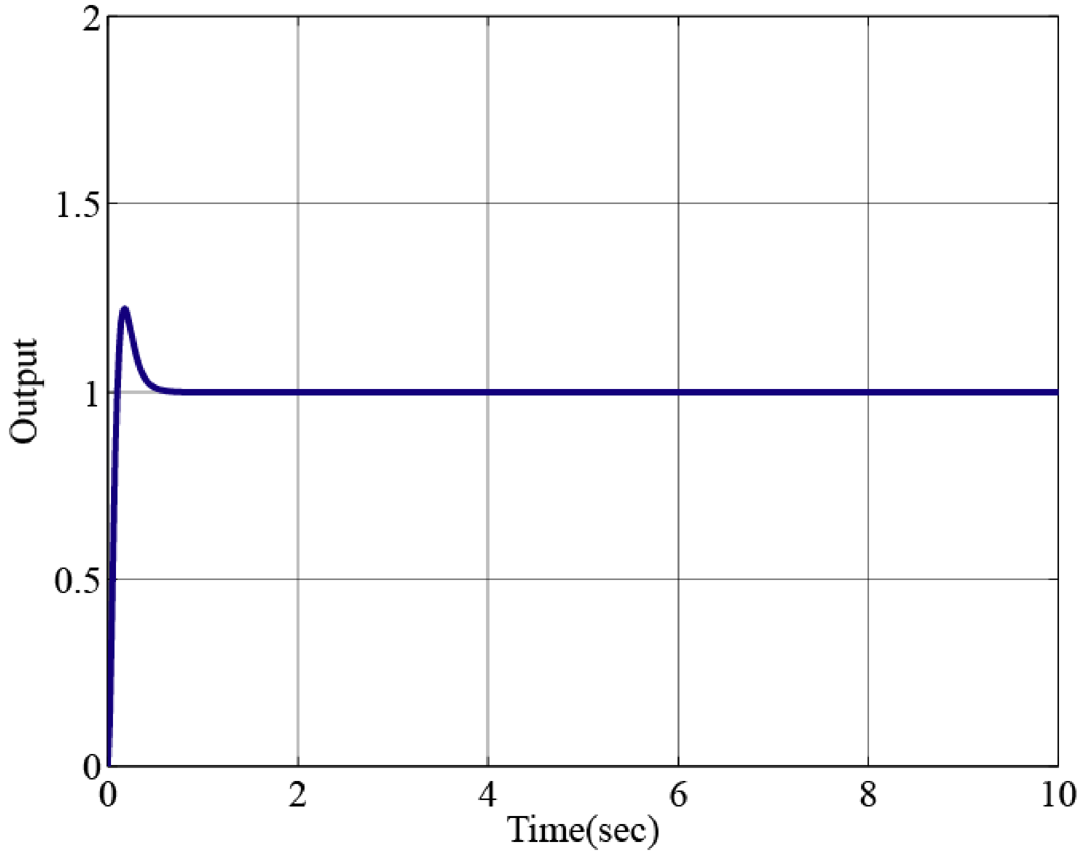
$$D(s) = K\frac{Ts+1}{\alpha Ts+1} = 100\frac{0.17s+1}{0.13(0.17s)+1}$$

예제: 진상 제어기



60
40
20
0
-20
-40
Magnitude (dB)
D(s) = 100 (0.17s + 1) / (0.13(0.17s) + 1)
D(s) = 100
10^-1
10^0
10^1
10^2
ω (rad/sec)
-90
-120
-150
-180
Phase (degree)
D(s) = 100 (0.17s + 1) / (0.13(0.17s) + 1)
D(s) = 100
10^-1
10^0
10^1
10^2
ω (rad/sec)

예제: 진상 제어기



MATLAB lead.m

```
num=100;
den=[1 1 0];
G=tf(num,den)

u=linspace(1,1,200);
t=linspace(0,10,200);

[y]=lsim(feedback(G,1),u,t);
figure(1)
plot(t,y);
grid on

w=logspace(0,3,200);
[mag,phase]=bode(num,den,w);
[gm,pm,wcg,wcp]=margin(G)
figure(2)
margin(num,den)
```

MATLAB lead.m

```
phimax=50;
alpha=(1-sin(pi*phimax/180))/(1+sin(pi*phimax/180))
10*log10(1/alpha)

[w' 20*log10(mag) phase ]

wmax=16.5;
T=1/(wmax*sqrt(alpha));
num1=[T 1];
den1=[T*alpha 1];
```

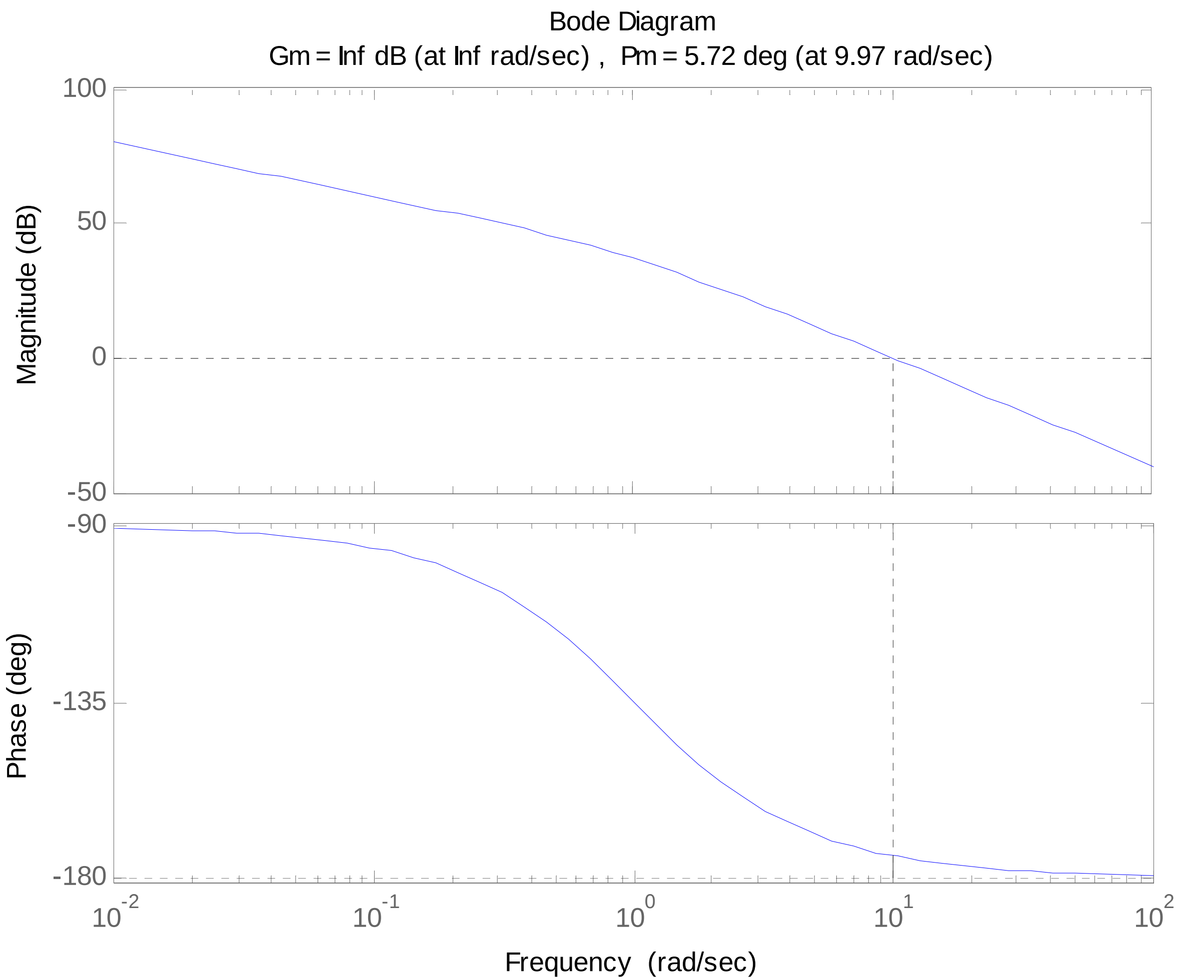
MATLAB lead.m

```
num=conv(num1,num);
den=conv(den1,den);
[mag,phase]=bode(num,den,w);
[gm,pm,wcg,wcp]=margin(mag,phase,w)
figure(3)
margin(num,den)

[y]=lsim(feedback(tf(num,den),1),u,t);
figure(4)
plot(t,y);
grid on

D=tf(num1,den1)
Dz=c2d(D,1/2000,'tustin')
```

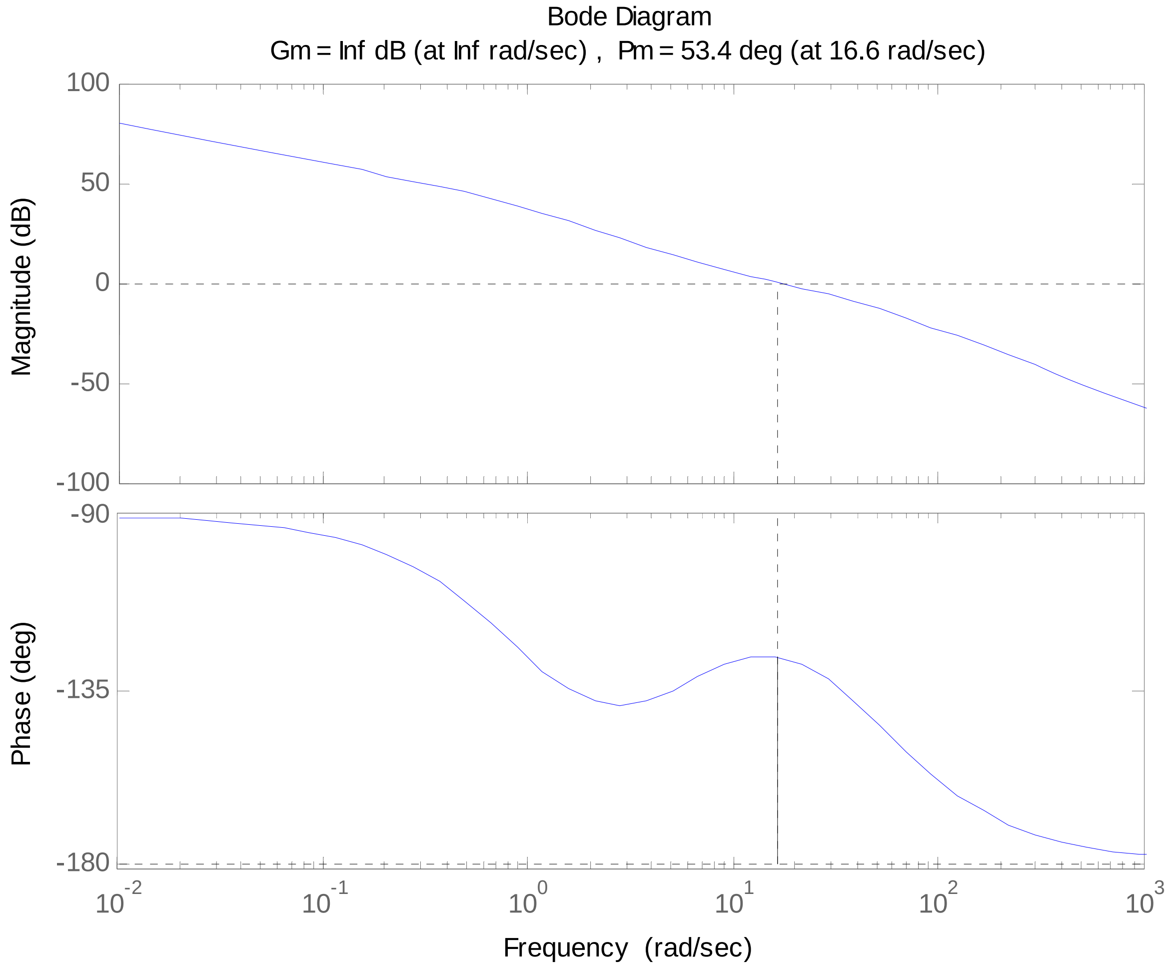
MATLAB lead.m

```
alpha =

  1.3247e-001


ans =

  8.7787e+000


ans =

  1.0000e+000  3.6990e+001 -1.3500e+002


  1.4481e+001 -6.4528e+000 -1.7605e+002
  1.4993e+001 -7.0545e+000 -1.7618e+002
  1.5522e+001 -7.6562e+000 -1.7631e+002
  1.6071e+001 -8.2580e+000 -1.7644e+002
  1.6638e+001 -8.8599e+000 -1.7656e+002
  1.7226e+001 -9.4618e+000 -1.7668e+002
```

MATLAB lead.m



MATLAB lead.m



MATLAB lead.m

- Digital Implementation

$$D(z) = \frac{U(z)}{E(z)} = K\frac{Ts+1}{\alpha Ts+1}\bigg|_{s=\frac{2}{T_s}\frac{z-1}{z+1}} = K\frac{T\frac{2}{T_s}\frac{z-1}{z+1}+1}{\alpha T\frac{2}{T_s}\frac{z-1}{z+1}+1}$$

$$u(k) = \frac{1}{T_s + 2\alpha T}\left[K\left(T_s + 2T\right)e(k) + K\left(T_s - 2T\right)e(k-1) + \left(2\alpha T - T_s\right)u(k-1)\right]$$

MATLAB lead.m

```
D=tf(num1,den1)
Dz=c2d(D,1/2000,'tustin')

Transfer function:
0.1665 s + 1
-------------
0.02206 s + 1


Transfer function:
7.475 z - 7.453
---------------
  z - 0.9776

Sampling time: 0.0005
```

Frequency Response of Digital System

$$Y(z) = G(z)U(z)$$

$$U(z) = \mathcal{Z}\left[\sin \omega t\right] = \frac{z \sin \omega T}{\left(z - e^{j\omega T}\right)\left(z - e^{-j\omega T}\right)}$$

$$Y(z) = \frac{G(z) z \sin \omega T}{\left(z - e^{j\omega T}\right)\left(z - e^{-j\omega T}\right)} = \frac{k_1 z}{z - e^{j\omega T}} + \frac{k_2 z}{z - e^{-j\omega T}} + Y_g(z)$$

$$Y_{ss}(z) = \frac{k_1 z}{z - e^{j\omega T}} + \frac{k_2 z}{z - e^{-j\omega T}}$$

$$k_1 = \frac{G(e^{j\omega T}) \sin \omega T}{e^{j\omega T} - e^{-j\omega T}} = \frac{G(e^{j\omega T})}{2j}$$

Frequency Response of Digital System

$$G(e^{j\omega T}) = \left| G(e^{j\omega T}) \right| e^{j\theta}, \theta = \measuredangle G(e^{j\omega T})$$

$$k_1 = \frac{\left| G(e^{j\omega T}) \right| e^{j\theta}}{2j}$$

$$k_2 = \frac{\left| G(e^{j\omega T}) \right| e^{-j\theta}}{-2j} = -\frac{\left| G(e^{j\omega T}) \right| e^{-j\theta}}{2j}$$

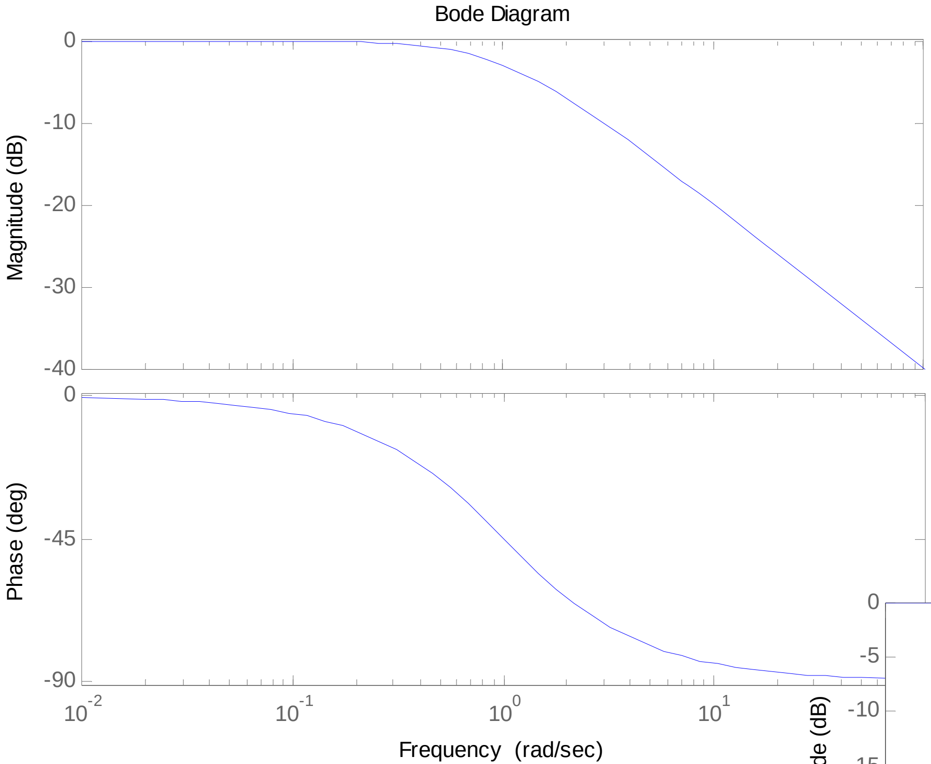
$$y_{ss}(kT) = k_1 (e^{j\omega T})^k + k_2 (e^{-j\omega T})^k = \left| G(e^{j\omega T}) \right| \sin(\omega kT + \theta)$$

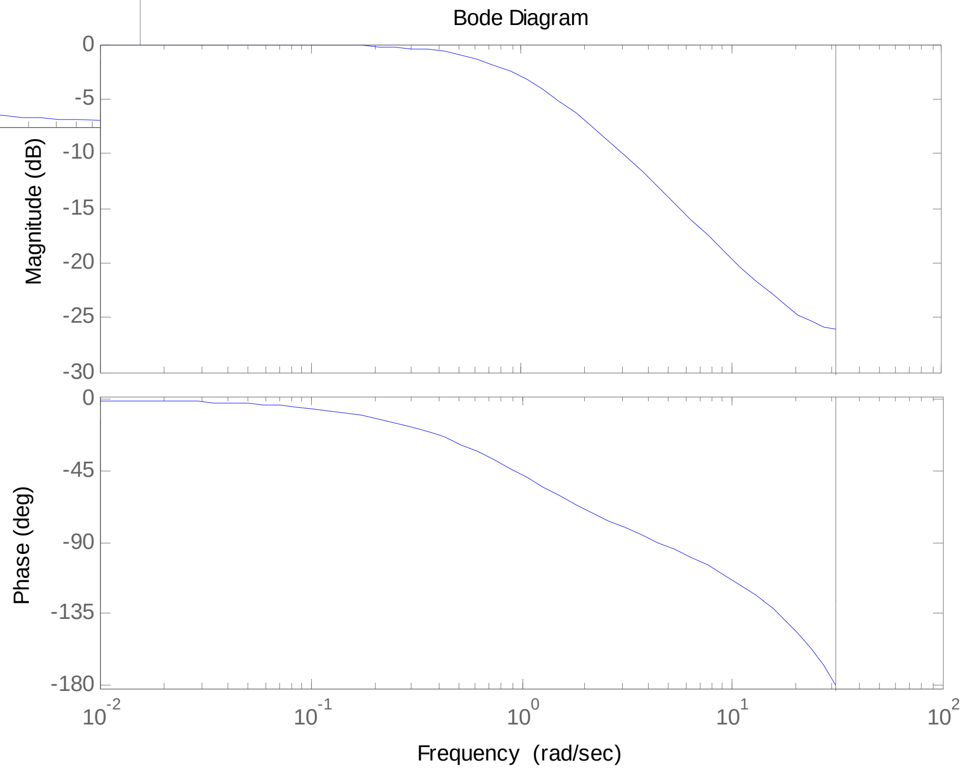
Frequency Response of Digital System

$$G(z) \rightarrow \left| G(e^{j\omega T}) \right|, \measuredangle G(e^{j\omega T})$$

$$G(s) \rightarrow \left| G(j\omega) \right|, \measuredangle G(j\omega)$$

```
>> G=tf(1,[1 1])

Transfer function:
  1
-----
s + 1

>> Gz=c2d(G,0.1)

Transfer function:
 0.09516
----------
z - 0.9048

Sampling time: 0.1
>> figure;bode(G)
>> figure;bode(Gz)
```


Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
Phase (deg)
0
-45
-90
10^-2
10^-1
10^0
10^1
Frequency (rad/sec)


Bode Diagram
Magnitude (dB)
0
-5
-10
-15
-20
-25
-30
Phase (deg)
0
-45
-90
-135
-180
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/sec)

Bilinear Transformation

$$z = \frac{1 + (T/2)w}{1 - (T/2)w}$$

$$w = \frac{2}{T}\frac{z-1}{z+1}$$

On the unit circle in z-plane: $z = e^{j\omega T}$

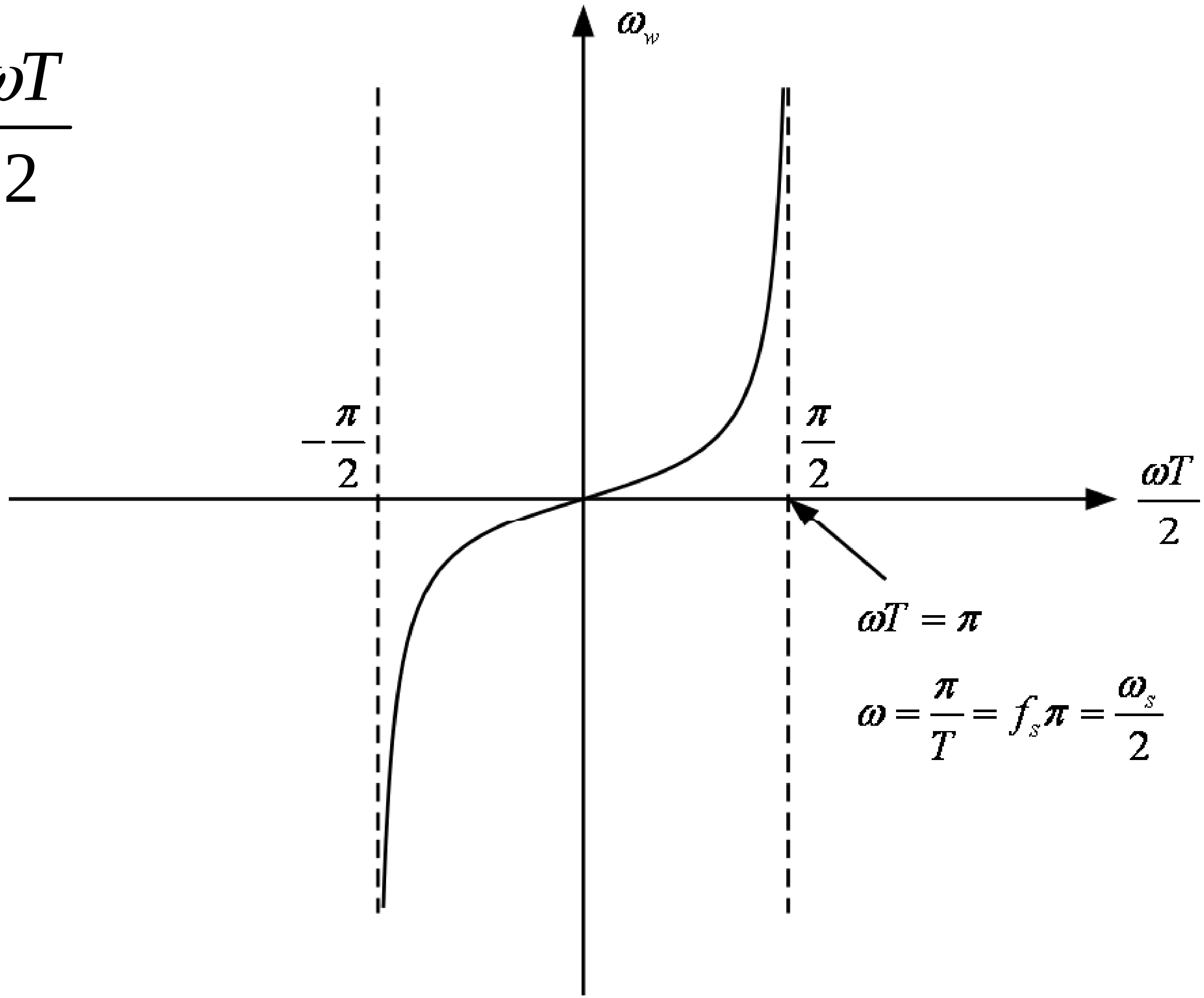
$$w = \frac{2}{T}\frac{z-1}{z+1}\bigg|_{z=e^{j\omega T}} = \frac{2}{T}\frac{e^{j\omega T}-1}{e^{j\omega T}+1} = \frac{2}{T}\frac{e^{j\omega T/2}-e^{-j\omega T/2}}{e^{j\omega T/2}+e^{-j\omega T/2}}$$

$$= j\frac{2}{T}\tan\frac{\omega T}{2} = j\omega_w$$

$$\omega_w = \frac{2}{T}\tan\frac{\omega T}{2}$$

Bilinear Transformation

$$\omega_w = \frac{2}{T}\tan\frac{\omega T}{2}$$



Bilinear Transformation

$$\omega << \omega_s$$

$$\omega_w = \frac{2}{T}\tan\frac{\omega T}{2} = \frac{2}{T}\frac{\sin\frac{\omega T}{2}}{\cos\frac{\omega T}{2}} \approx \frac{2}{T}\frac{\omega T}{2} = \omega$$

$$\frac{\omega T}{2} < \frac{\pi}{10} \rightarrow \text{error is less than 4\%}$$

$$\omega < \frac{2\pi}{10T} = \frac{\omega_s}{10}$$

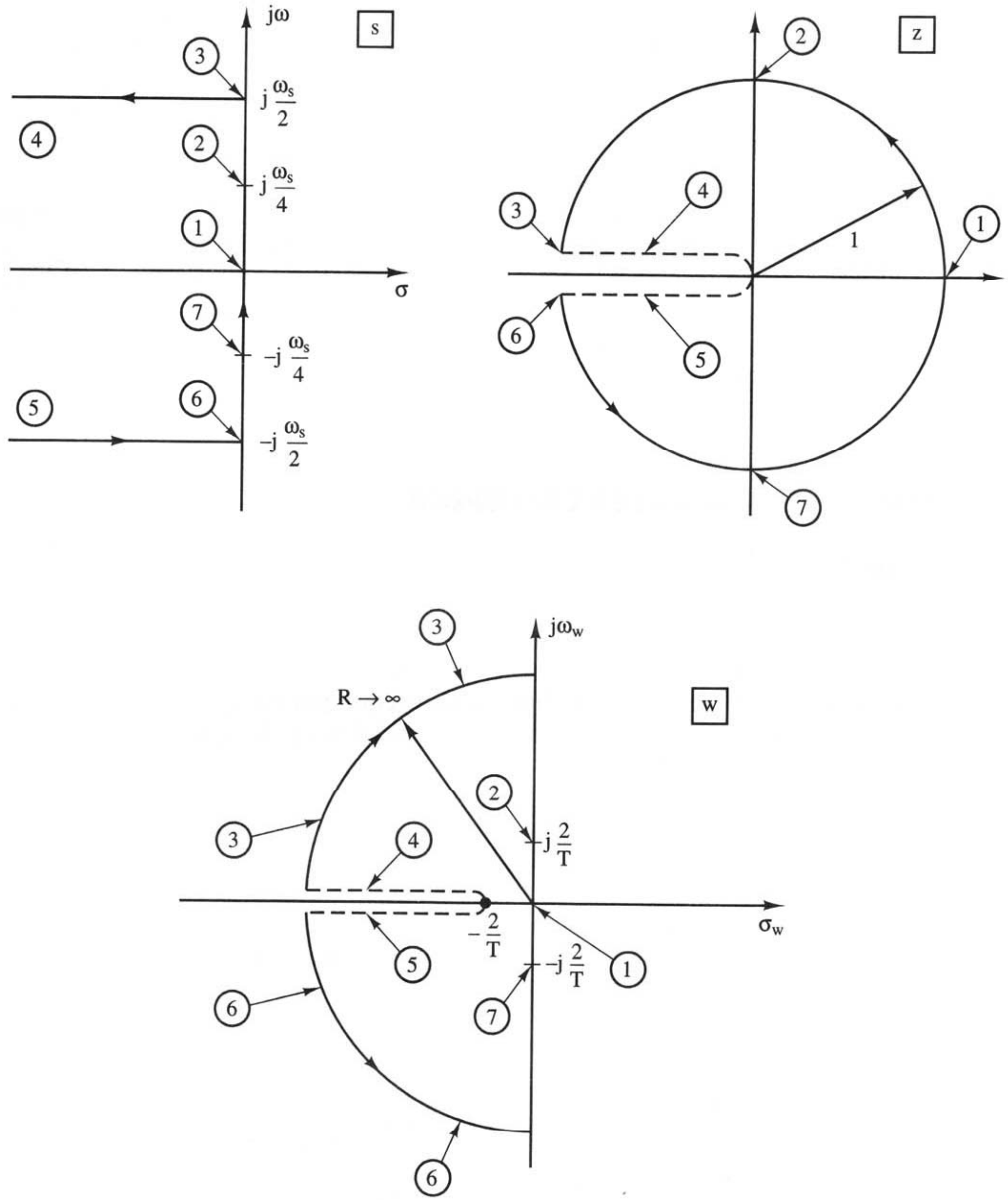


Figure 7-4 Mapping from s-plane to z-plane to w-plane.

MATLAB lead_digital.m

```
N=200;
Ts=1/100
Gp=tf(100,[1 1 0])
Gz=c2d(Gp,Ts,'zoh')

figure(1)
step(feedback(Gz,1),10)

Gw=d2c(Gz,'tustin')
figure(2)
margin(Gw);

w=logspace(-2,3,N);
[mag,phase]=bode(Gw,w);
for i=1:N
   Gw_mag(i)=mag(i);
   Gw_phase(i)=phase(i);
end
```

MATLAB lead_digital.m

```
phimax=50;
alpha=(1-sin(pi*phimax/180))/(1+sin(pi*phimax/180))
10*log10(1/alpha)

[w' (20*log10(Gw_mag))'  Gw_phase']

wmax=16.5;
T=1/(wmax*sqrt(alpha));
num1=[T 1];
den1=[T*alpha 1];

Dw=tf(num1,den1)

figure(3)
margin(Dw*Gw)

Dz=c2d(Dw,Ts,'tustin')
figure(4)
step(feedback(Dz*Gz,1),10)
```

MATLAB lead_digital.m

```
Transfer function:
  100
-------
s^2 + s


Transfer function:
0.004983 z + 0.004967
---------------------
 z^2 - 1.99 z + 0.99

Sampling time: 0.01

Transfer function:
-4.167e-006 s^2 - 0.4992 s + 100
--------------------------------
     s^2 + s + 1.11e-013
```

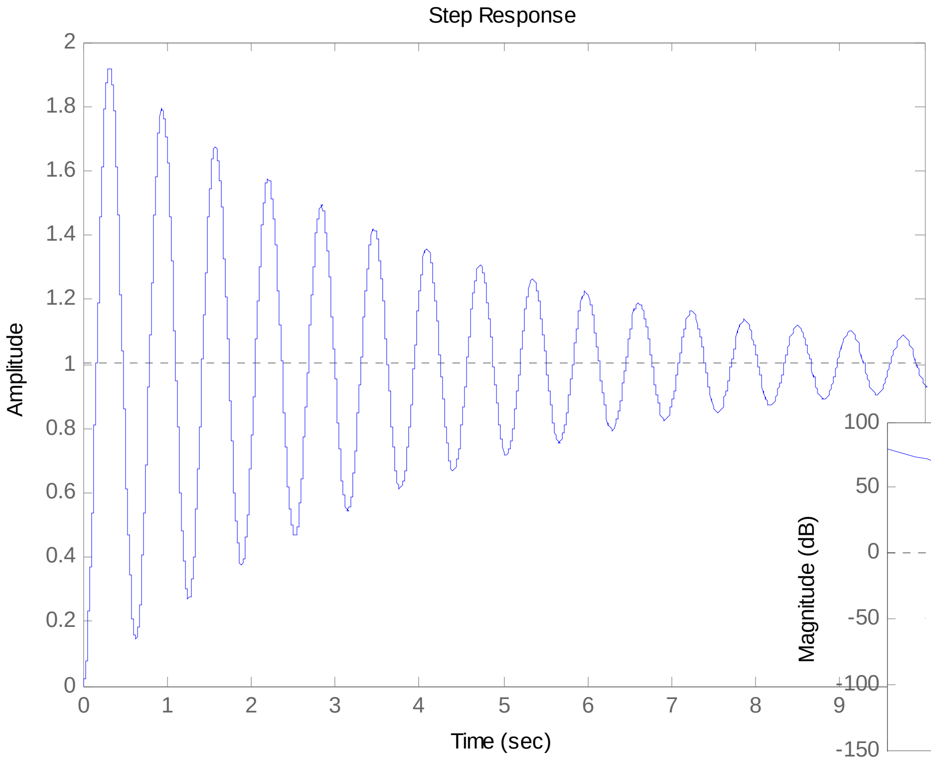
MATLAB lead_digital.m

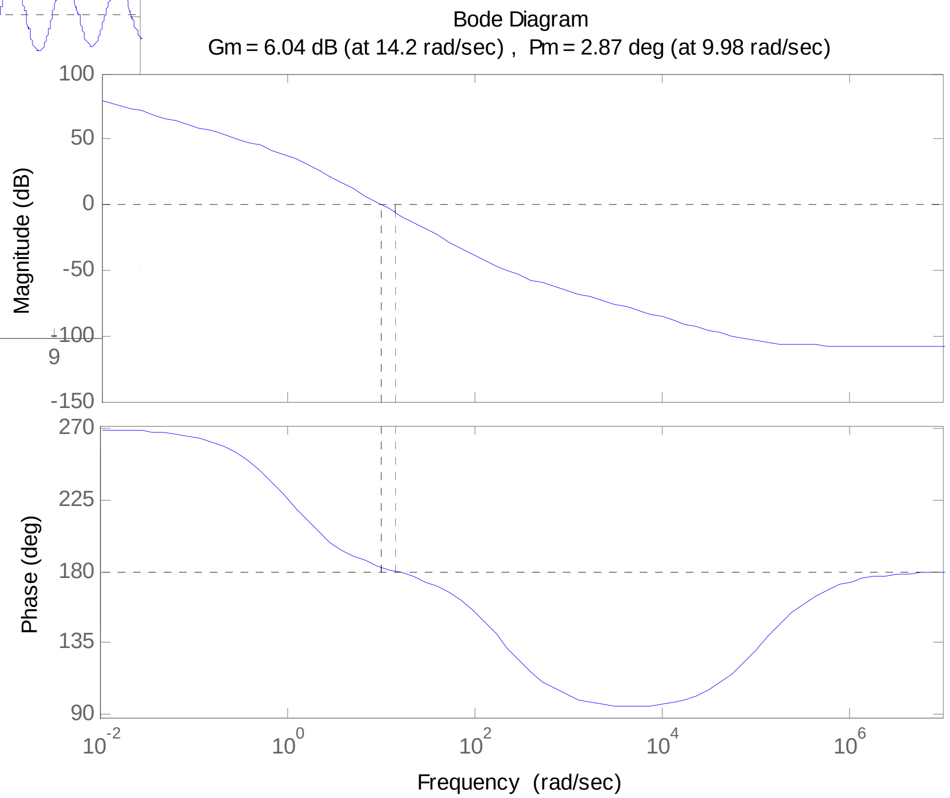
```
alpha =

  1.3247e-001


ans =

  8.7787e+000


ans =

  1.0000e-002  8.0000e+001 -9.0576e+001


  1.4650e+001 -6.6302e+000 -1.8028e+002
  1.5522e+001 -7.6302e+000 -1.8074e+002
  1.6447e+001 -8.6300e+000 -1.8121e+002
  1.7426e+001 -9.6297e+000 -1.8169e+002
```

MATLAB lead_digital.m

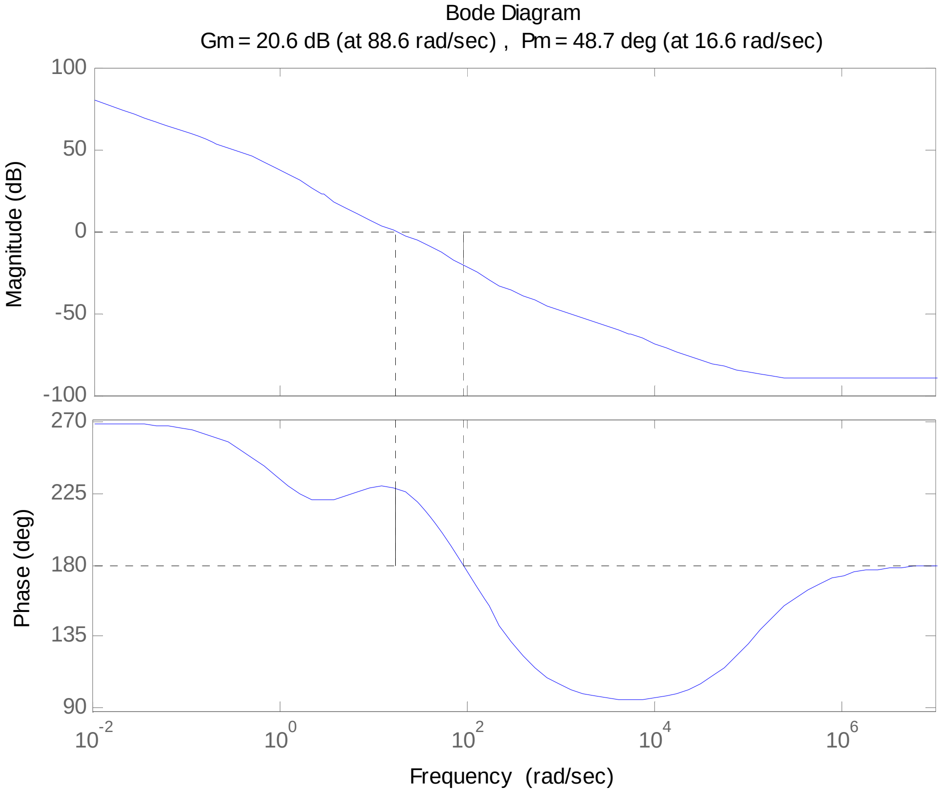
```
Transfer function:
0.1665 s + 1
-------------
0.02206 s + 1


Transfer function:
6.339 z - 5.969
---------------
  z - 0.6304

Sampling time: 0.01
```

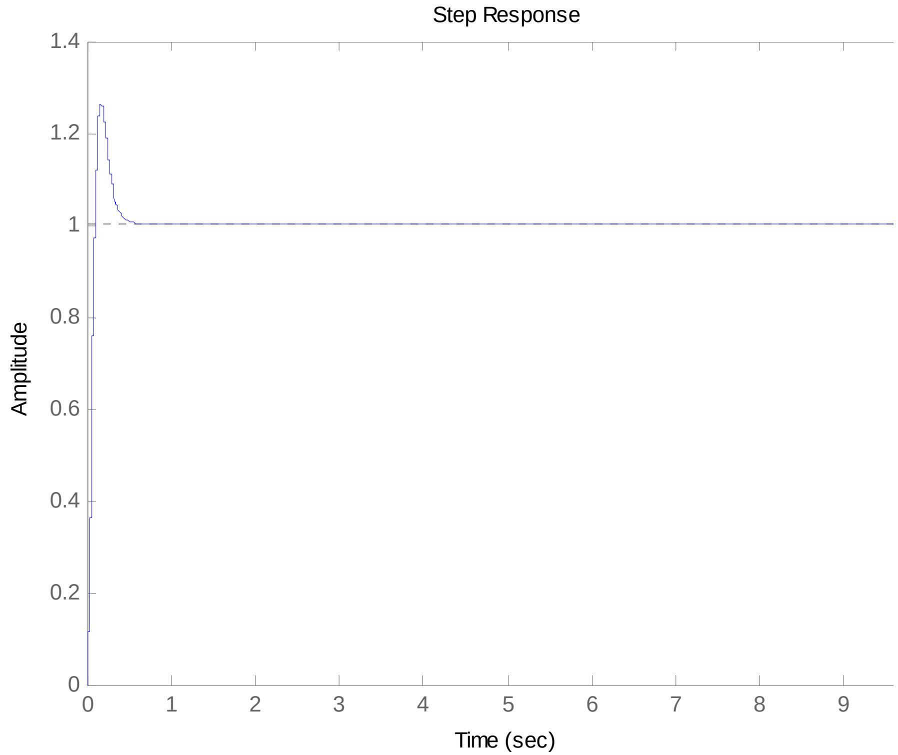
MATLAB lead_digital.m





MATLAB lead_digital.m





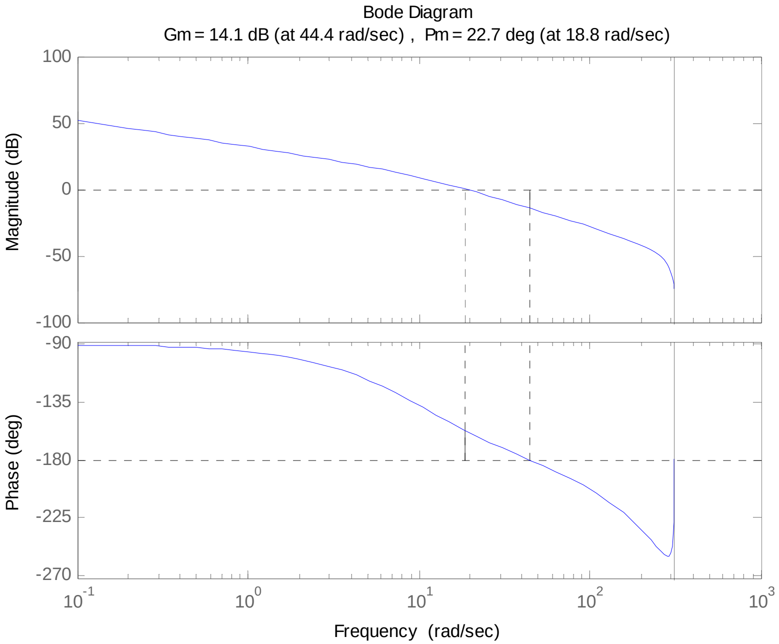
Gz 와 Gw 의 GM 과 PM

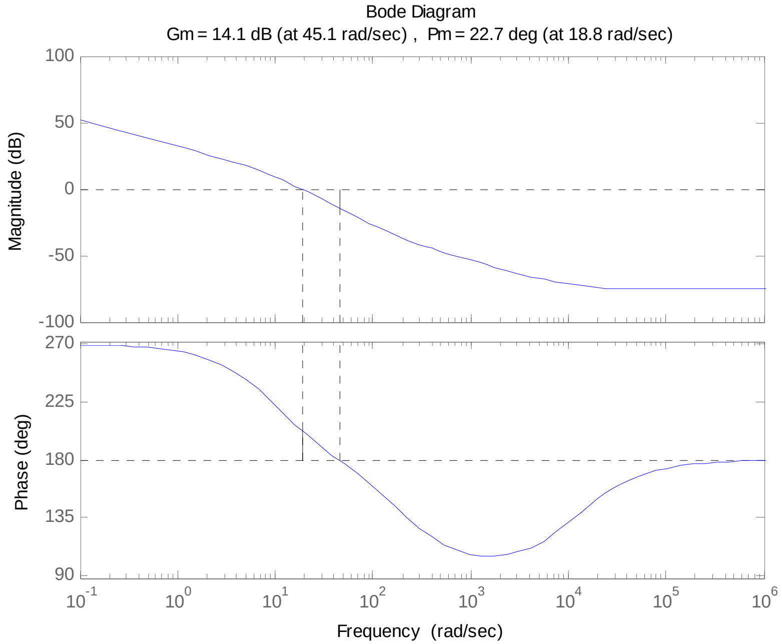
```
N=200;
Ts=1/100
Gp=tf(40,[0.1 1 0])
Gz=c2d(Gp,Ts,'zoh')

figure(1)
margin(Gz);

Gw=d2c(Gz,'tustin')
figure(2)
margin(Gw);

w=logspace(-2,2,N);
[mag,phase]=bode(Gw,w);
for i=1:N
    Gw_mag(i)=mag(i);
    Gw_phase(i)=phase(i);
end
```

Gz 와 Gw 의 GM 과 PM

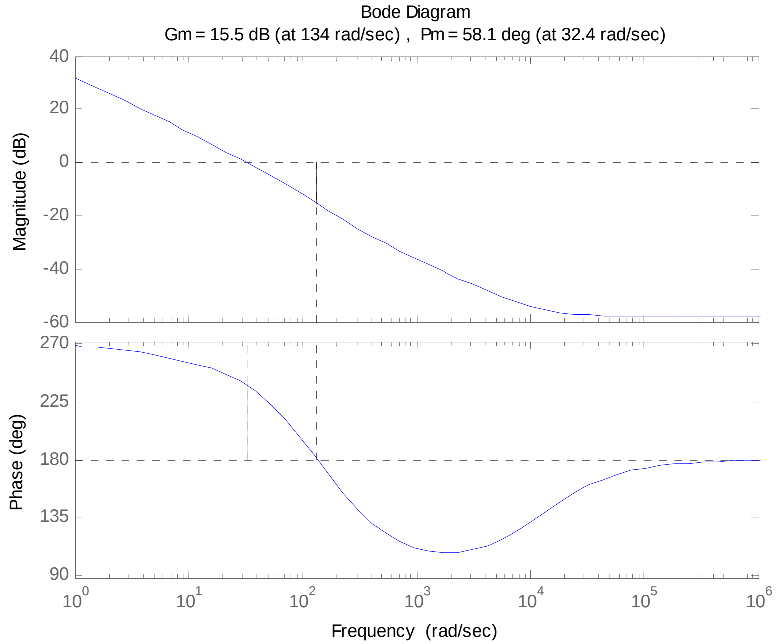
```
phimax=50;
alpha=(1-sin(pi*phimax/180))/(1+sin(pi*phimax/180))
10*log10(1/alpha)

[w' (20*log10(Gw_mag))'  Gw_phase']

wmax=33;
T=1/(wmax*sqrt(alpha));
num1=[T 1];
den1=[T*alpha 1];

Dw=tf(num1,den1)

figure(3)
margin(Dw*Gw)

Dz=c2d(Dw,Ts,'tustin')
figure(4)
margin(Dz*Gz)
```

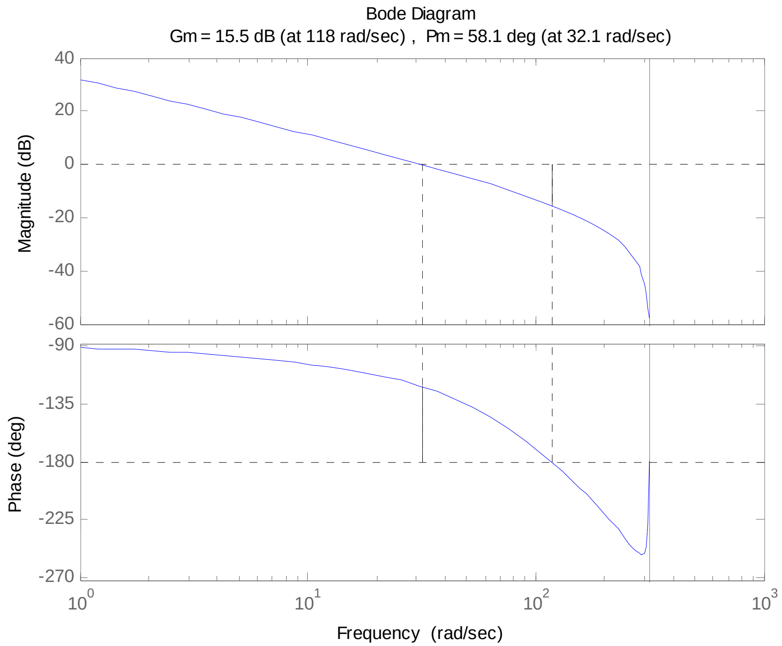
Gz 와 Gw 의 GM 과 PM





Gz 와 Gw 의 GM 과 PM





$$z = \frac{1 + (T/2)w}{1 - (T/2)w}$$

$$w = \frac{2}{T}\frac{z - 1}{z + 1}$$

On the unit circle in z-plane: $z = e^{j\omega T}$

$$w = \frac{2}{T}\frac{z - 1}{z + 1}\bigg|_{z = e^{j\omega T}} = \frac{2}{T}\frac{e^{j\omega T} - 1}{e^{j\omega T} + 1} = \frac{2}{T}\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{e^{j\omega T/2} + e^{-j\omega T/2}}$$

$$= j\frac{2}{T}\tan\frac{\omega T}{2} = j\omega_w$$

$$\omega_w = \frac{2}{T}\tan\frac{\omega T}{2}$$

$$e^{j\omega T} = \frac{1 + (T/2)j\omega_w}{1 - (T/2)j\omega_w}$$

$$G_w(w) = G(z)\Big|_{z = \frac{1 + (T/2)w}{1 - (T/2)w}} = G\left(\frac{1 + (T/2)w}{1 - (T/2)w}\right)$$

$$G_w(j\omega_w) = G\left(\frac{1 + (T/2)j\omega_w}{1 - (T/2)j\omega_w}\right) = G(e^{j\omega T})$$