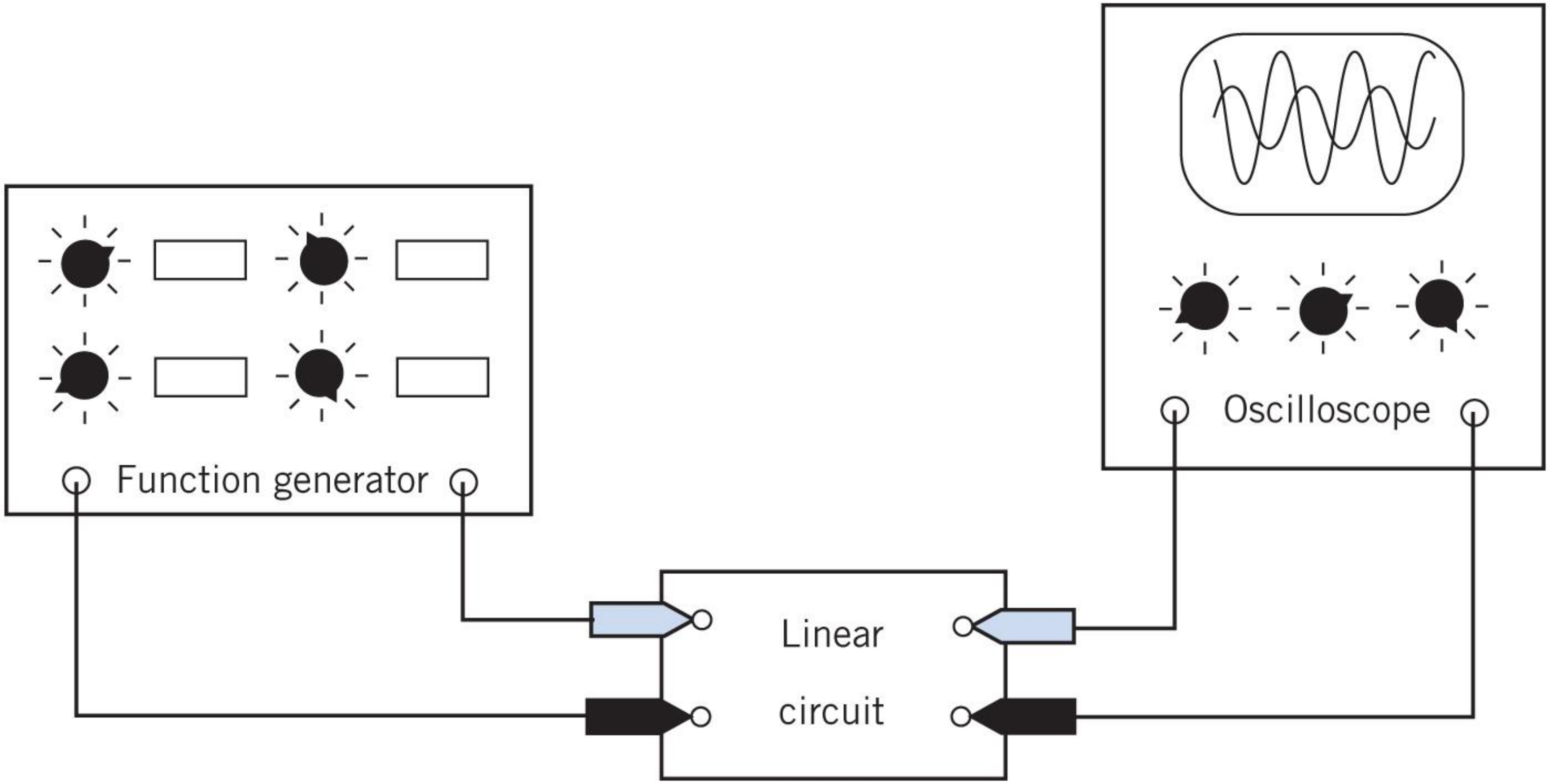
Frequency Response

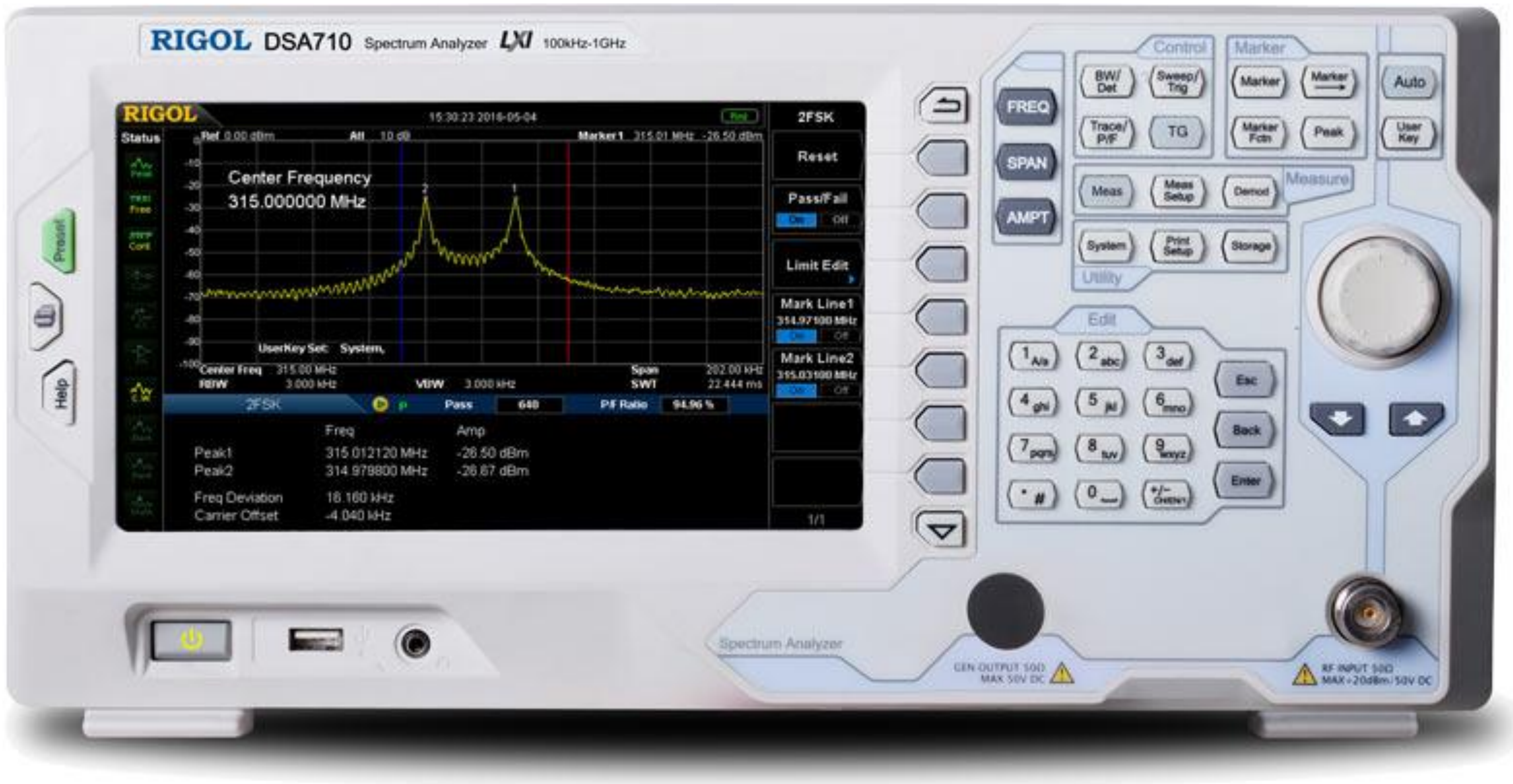
RC & RLC Circuits

Measuring Responses

- Define input and output
- Time response and frequency response

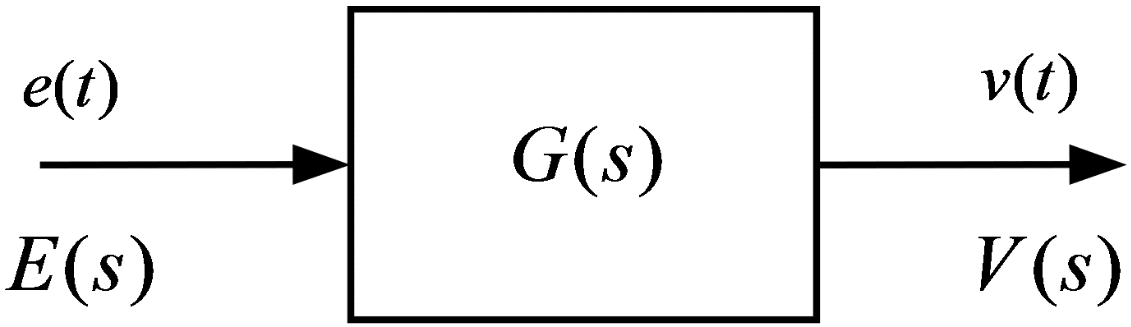


Spectrum Analyzer



Frequency Response

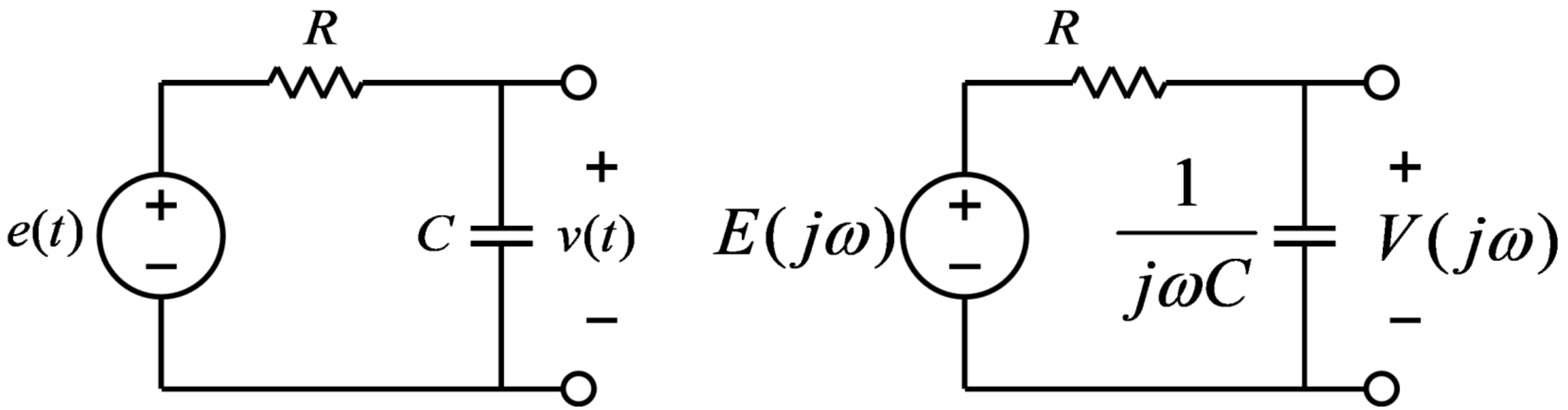
- Sinusoidal Input, Steady State



$$e(t) = A\cos(\omega t)$$

$$v(t) = A\left[\qquad\right]\cos\left(\omega t + \left[\qquad\right]\right)$$

Network Function

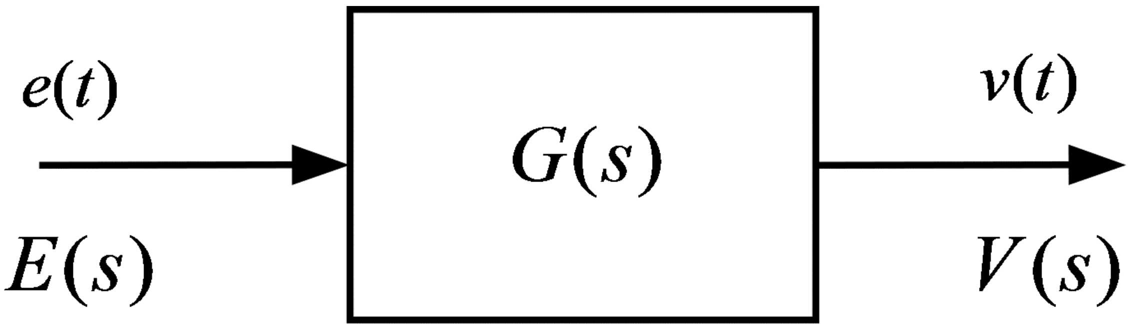


$$V(j\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} E(j\omega) = \frac{1}{j\omega RC + 1} E(j\omega)$$

$$G(j\omega) = \frac{V(j\omega)}{E(j\omega)} = \frac{1}{j\omega RC + 1}$$

Frequency Response

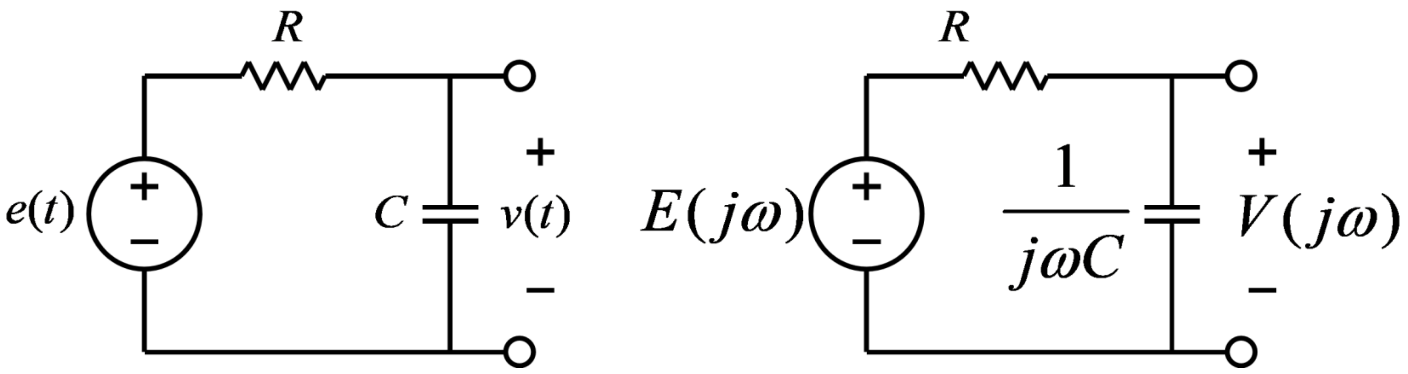
- Sinusoidal Input, Steady State



$$e(t) = A\cos(\omega t)$$

$$v(t) = A\left|G(j\omega)\right|\cos\left(\omega t + \measuredangle G(j\omega)\right)$$

Low Pass RC Circuit

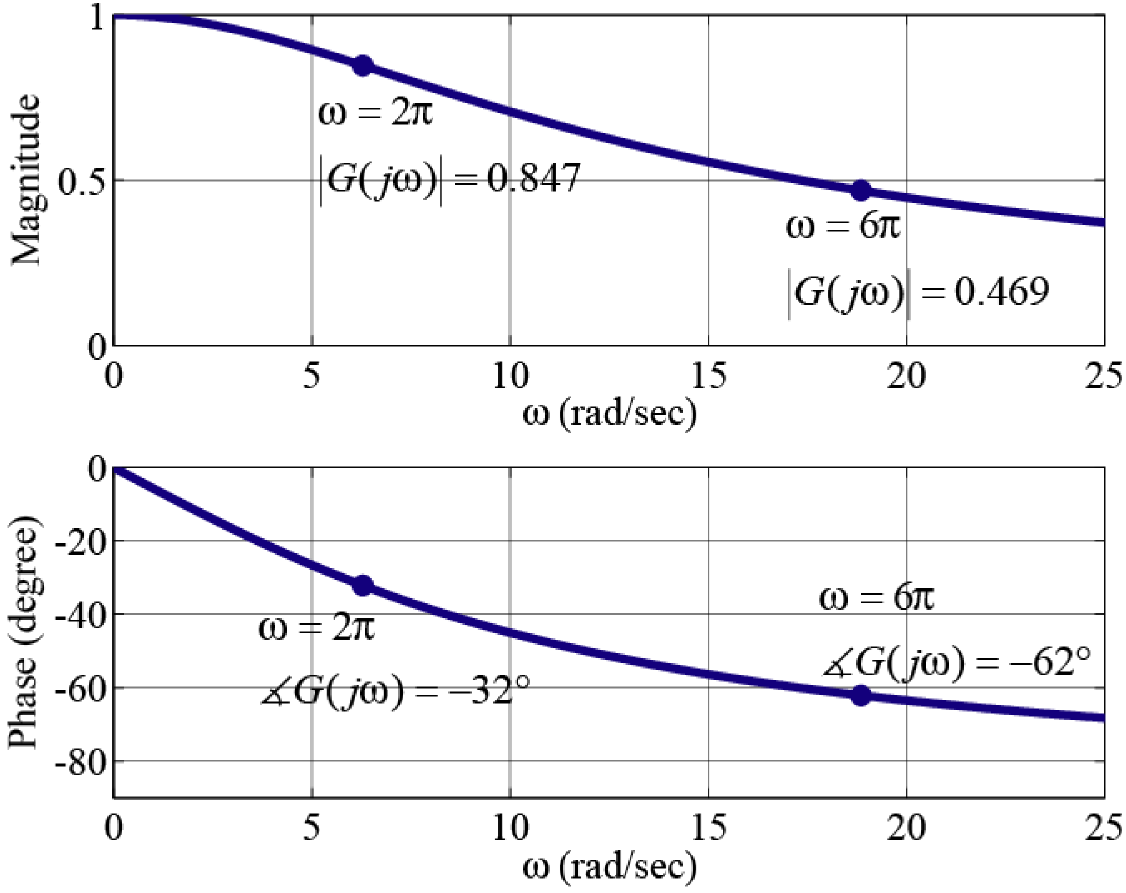


$$G(j\omega) = \frac{V(j\omega)}{E(j\omega)} = \frac{1}{j\omega RC + 1}$$

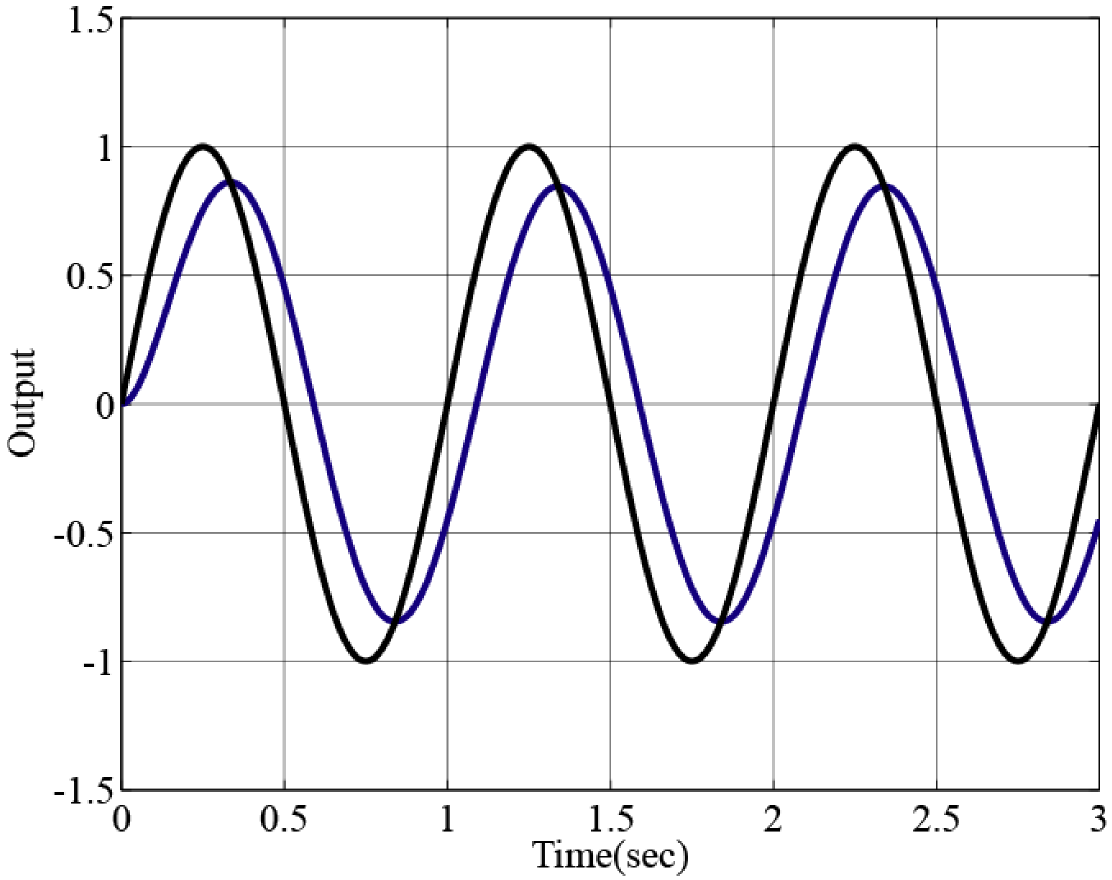
$$\left|G(j\omega)\right| = \left|\frac{1}{j\omega RC + 1}\right| = \frac{1}{\sqrt{(\omega RC)^2 + 1}}$$

$$\measuredangle G(j\omega) = \measuredangle\left(\frac{1}{j\omega RC + 1}\right) = -\tan^{-1}(\omega RC)$$

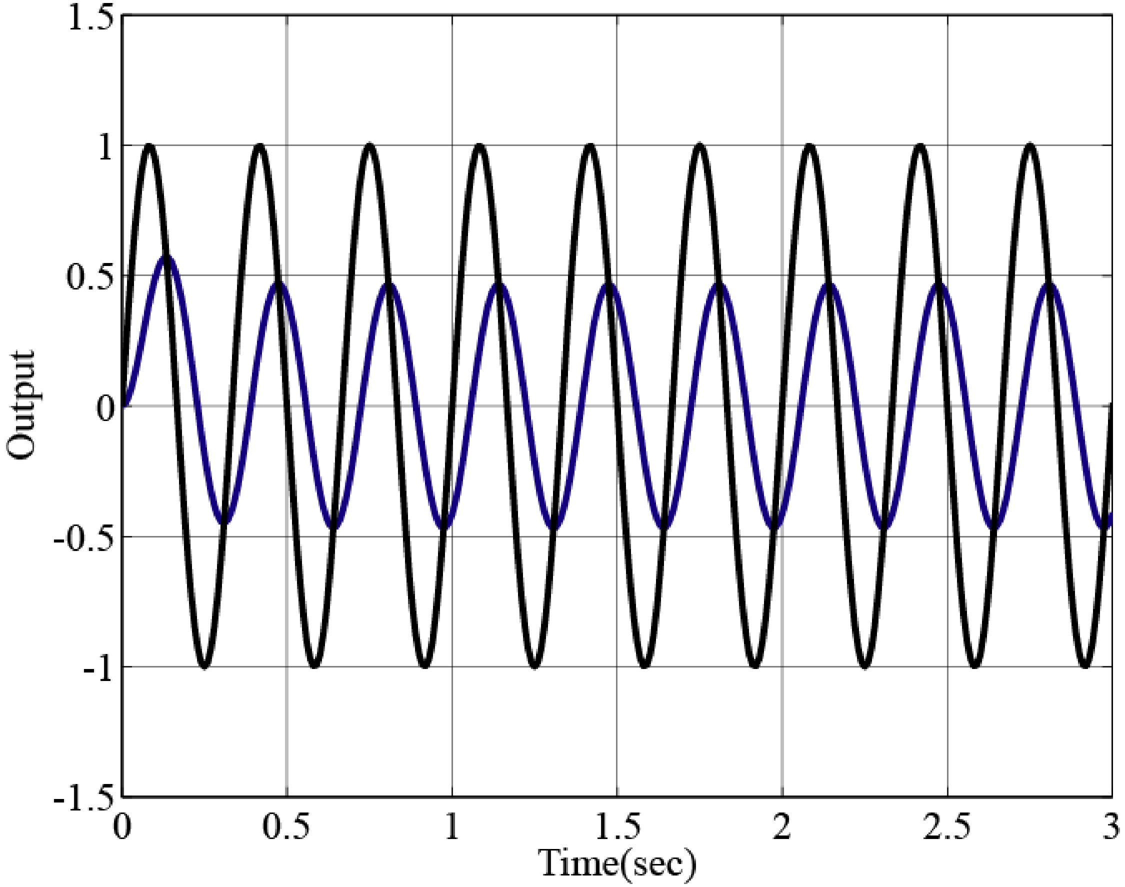
$$R = 100\Omega, C = 0.001F$$



1Hz

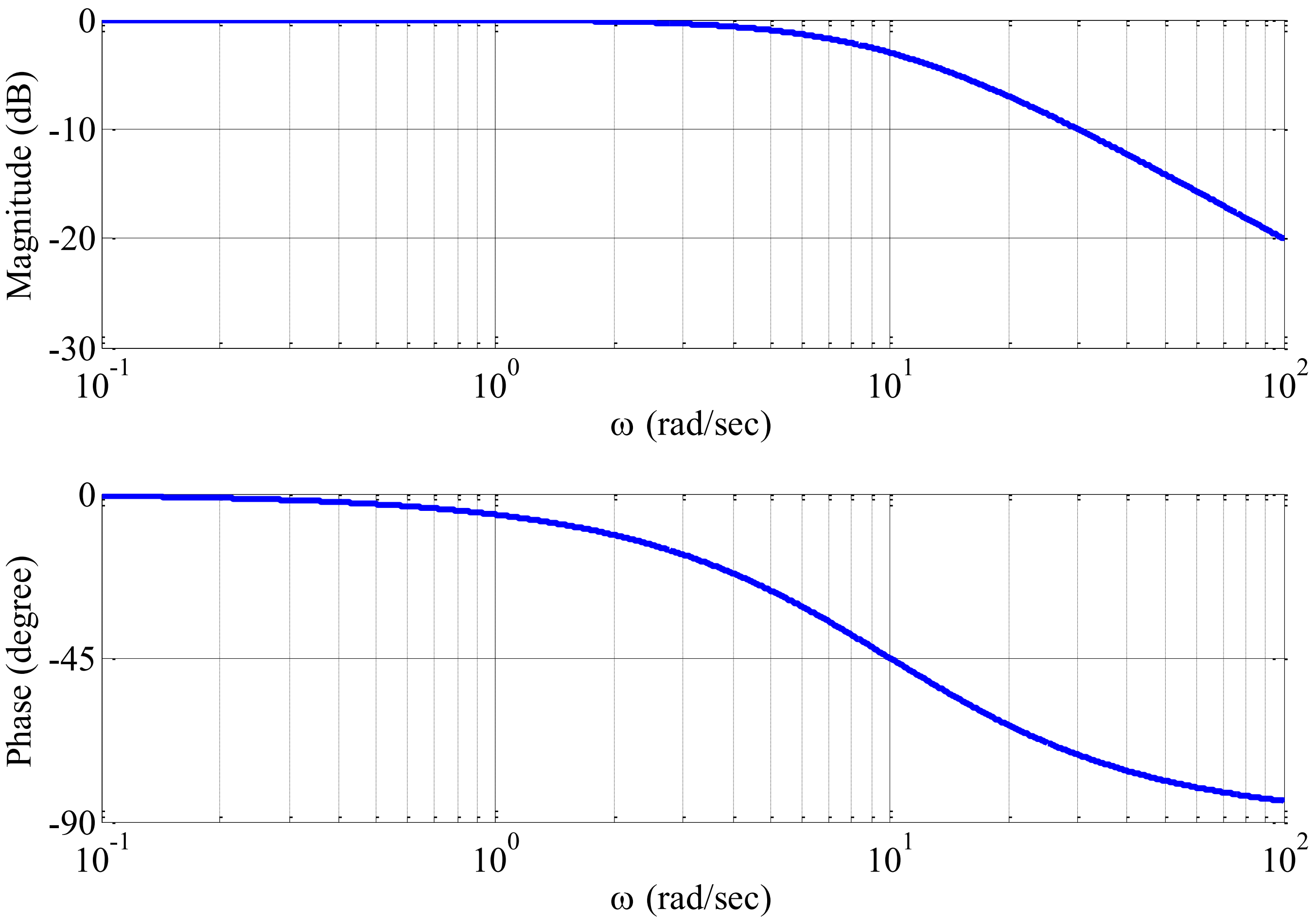


3Hz



Bode Plot

- dB(deci Bell): 20log(magnitude)



Band Width, Cut Off Frequency

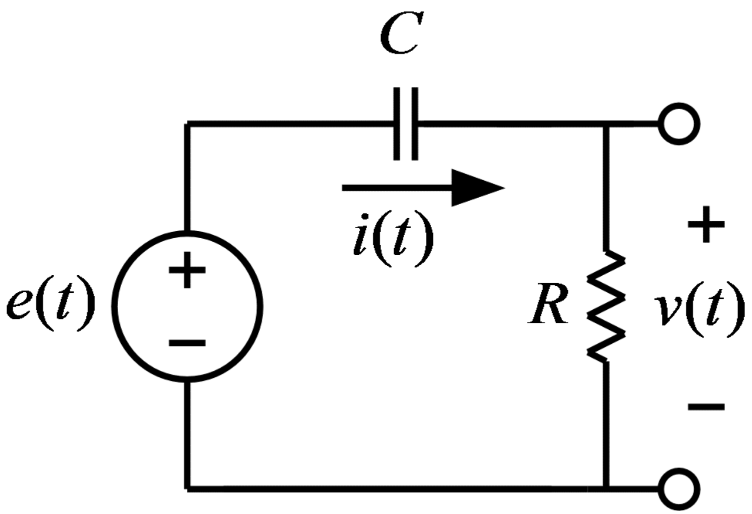
$$G(j\omega) = \frac{V(j\omega)}{E(j\omega)} = \frac{1}{j\omega RC + 1}$$

$$\left|G(j\omega)\right| = \left|\frac{1}{j\omega RC + 1}\right| = \frac{1}{\sqrt{\left(\omega RC\right)^2 + 1}}$$

$$\omega RC = 1 \Rightarrow \left|G(j\omega)\right| = \left|\frac{1}{j+1}\right| = \frac{1}{\sqrt{2}} \Rightarrow 20\log_{10}\frac{1}{\sqrt{2}} = -3dB$$

$$\omega = \frac{1}{RC}$$

High Pass RC Circuit



$$V(j\omega) = \frac{R}{R + \frac{1}{j\omega C}} E(j\omega) = \frac{j\omega RC}{j\omega RC + 1} E(j\omega)$$

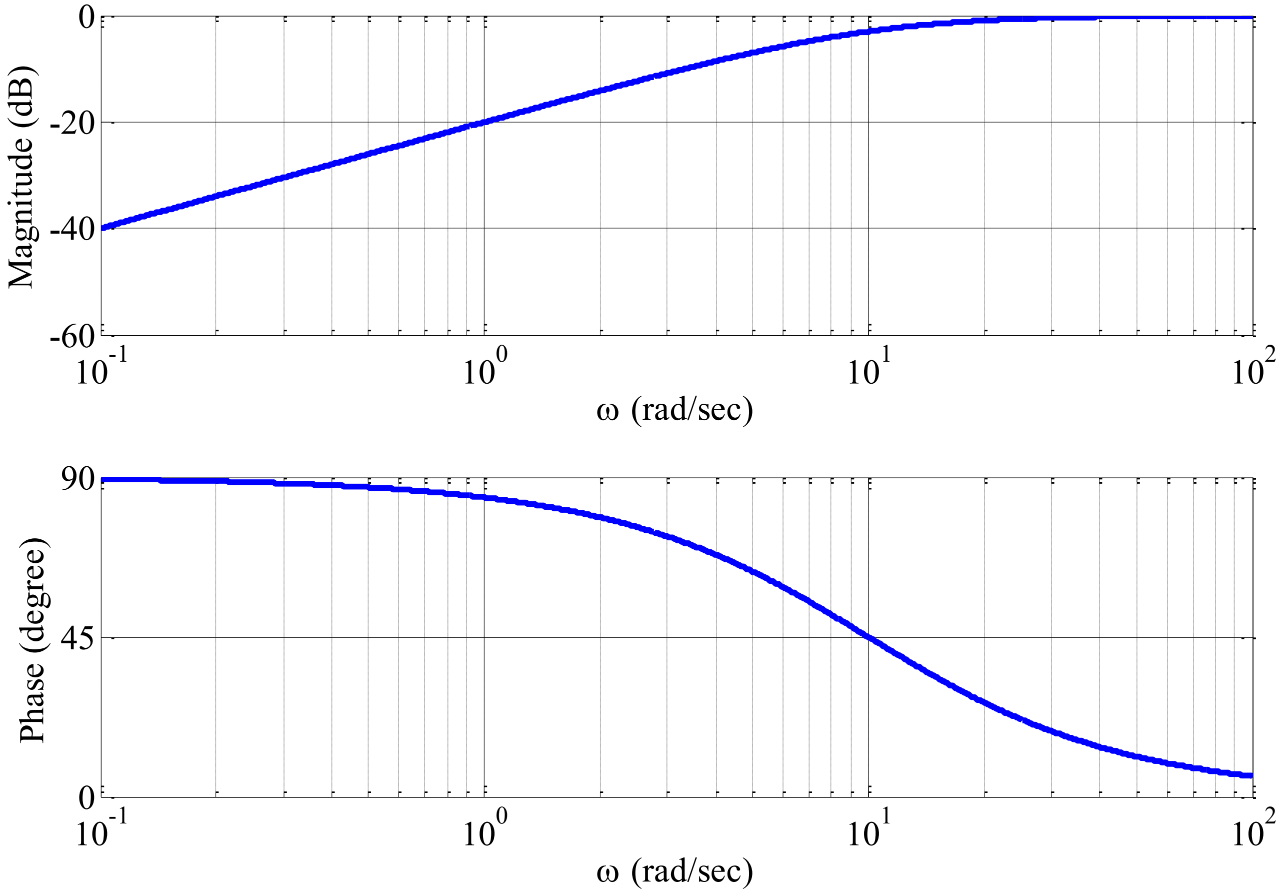
$$G(j\omega) = \frac{V(j\omega)}{E(j\omega)} = \frac{j\omega RC}{j\omega RC + 1}$$

$$G(j\omega) = \frac{V(j\omega)}{E(j\omega)} = \frac{j\omega RC}{j\omega RC + 1}$$

$$\left|G(j\omega)\right| = \left|\frac{j\omega RC}{j\omega RC + 1}\right| = \frac{\omega RC}{\sqrt{\left(\omega RC\right)^2 + 1}}$$

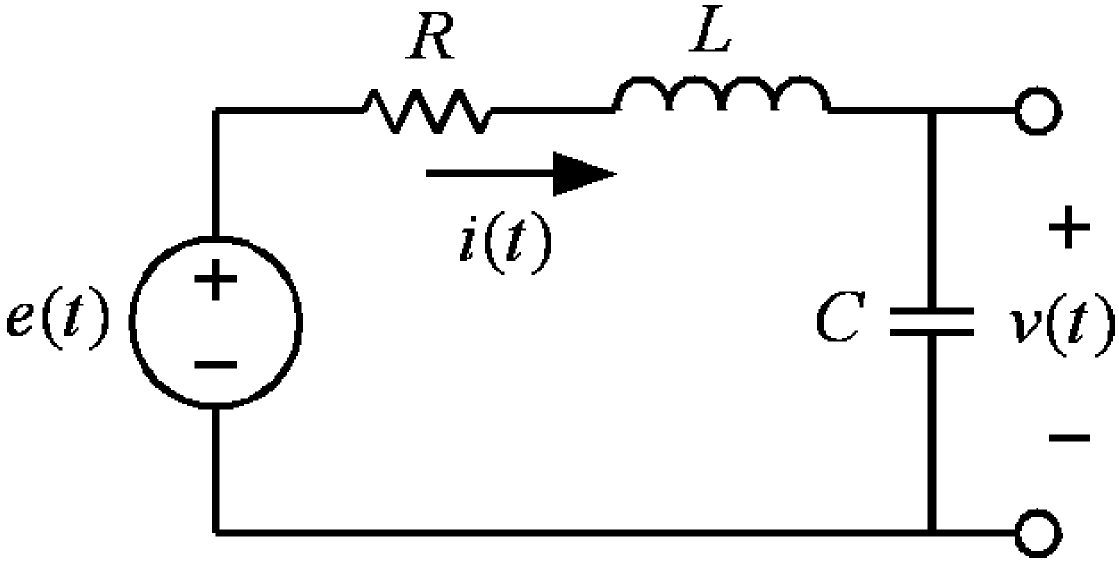
$$\measuredangle G(j\omega) = \measuredangle\left(\frac{1}{j\omega RC + 1}\right) = 90^\circ - \tan^{-1}\left(\omega RC\right)$$

Bode Plot



0
-20
-40
-60
Magnitude (dB)
10^{-1}
10^{0}
10^{1}
10^{2}
ω (rad/sec)
90
45
0
Phase (degree)
10^{-1}
10^{0}
10^{1}
10^{2}
ω (rad/sec)

Low Pass RLC Circuit



$$V(j\omega) = \frac{\dfrac{1}{j\omega C}}{j\omega L + R + \dfrac{1}{j\omega C}} E(j\omega) = \frac{1}{(j\omega)^2 LC + (j\omega)RC + 1} E(j\omega)$$

Low Pass RLC Circuit

$$V(j\omega) = \frac{\frac{1}{j\omega C}}{j\omega L + R + \frac{1}{j\omega C}} E(j\omega) = \frac{1}{(j\omega)^2 LC + (j\omega)RC + 1} E(j\omega)$$

$$G(j\omega) = \frac{V(j\omega)}{E(j\omega)} = \frac{1}{(j\omega)^2 LC + (j\omega)RC + 1} = \frac{1/(LC)}{(j\omega)^2 + (R/L)(j\omega) + 1/(LC)}$$

$$G(j\omega) = \frac{\omega_0^2}{(j\omega)^2 + 2\alpha(j\omega) + \omega_0^2}$$

$$\alpha = \frac{R}{2L}, \omega_0 = \frac{1}{\sqrt{LC}}$$

$$R = 100\Omega, L = 100mH, C = 0.1\mu F$$